PROCEEDINGS
THE INSTITUTION OF CIVIL ENGINEERS

PART I
JANUARY 1954

ORDINARY MEETING

3 November, 1953

HENRY FRANCIS CRONIN, C.B.E., M.C., B.Sc.(Eng.), the retiring President, in the Chair

The Council reported that they had recently transferred to the class of

Members

ALLAN, CHARLES LEWIS CUTHBERT, B.A. (*Cantab.*).
ARBUTHNOTT, ROBERT.
ARUMUGAM, SANMUGAM, B.Sc.(Eng.) (*London*).
BAKER, LESLIE HAROLD, B.Sc.(Eng.) (*London*).
BAXTER, GEORGE RONALD.
BHIDE, KASHINATH RAMCHANDRA, B.E. (*Bombay*).
BOLTON, JOHN, M.Eng. (*Liverpool*).
BOWEN, FRANK MAURICE.
BUCHAN, DAVID WADDELL, B.Sc. (*Glasgow*).
BULMAN, WILLIAM ELLERINGTON, B.Sc. (Eng.) (*London*).
CAMPBELL, JOSEPH GARDEN.
COLEMAN, GILBERT PEDDIE, B.Sc.(Eng.) (*London*).
DESAI, DAHYABHAI SHIVABHAI, B.Eng. (*Sheffield*).
DIXON, JOHN MARTIN.
FAIRKINER, RICHARD HONE, B.A., B.A.I. (*Dublin*).
FRASER, JAMES MCLAREN, B.Sc. (*Edinburgh*).
GANDELL, ALAN THOMAS.
GEDDES, WILLIAM GEORGE NICHOLSON, B.Sc. (*Edinburgh*).
GREEN, CHARLES MACLACHLAN, B.Sc. (*Glasgow*).
HARRIS, CLIFFORD, B.Sc. (*Cape Town*).
HASWELL, JOHN HENRY RODERICK, B.Sc. (Eng.) (*London*).
HOROWITZ, LUDWIG.
HVISTENDAHL, HERBERT SELWYN, B.Sc. (Eng.) (*London*).
HYATT, KENNETH EDWIN, B.Sc.(Eng.) (*London*).
JOHNSON, RICHARD, B.Eng. (*Sheffield*).
JORDAN, SYDNEY KENNETH.
KEEFE, HAROLD GEORGE.
KELLEY, HERBERT HARDING.
LANDER, JOHN HELIER, O.B.E., M.A. (*Cantab.*).
LEE, JOHN JAMES.
MACDONALD, RODERICK HUGH, M.A. (*Cantab.*).
MAGUIRE, EDMUND CLAUDE, B.Sc. (*Durham*).
MALLAGH, TERENCE JOHN SLOAN, M.A., M.A.I. (*Dublin*).
MASON, DAVID ERSKINE SINCLAIR.
MILLEDGE, ROYDON ARTHUR.
MORRIS, ALFRED THOMAS.
MUFF, JOHN RAYMOND.
NELSON, HENRY GEORGE, M.A. (*Cantab.*).
PARKER, DONALD, B.Sc. (*Birmingham*).

1

Ratcliffe, Edward William, M.Eng. (*Liverpool*).
Rowntree, Norman Andrew Forster, B.Sc.(Eng.) (*London*).
Shiell, James Wyllie, B.Sc. (*Edinburgh*).
Singleton-Green, John, M.Sc. (*Manchester*).
Todman, Daniel Innes.
Watermeyer, Thomas Christian, B.Sc. (*Cape Town*).

and had admitted as

Graduates

Abbot, Eric Norman Vincent, B.Sc. (Eng.) (*London*).
Aizlewood, Herbert, B.Sc. (*Birmingham*), Stud.I.C.E.
Amesbury, Kenneth, B.Sc. (*Bristol*), Stud.I.C.E.
Amphlet, Ronald George, Stud.I.C.E.
Armstrong, David John, B.A. (*Cantab.*), Stud.I.C.E.
Armstrong-Wright, Alan Thomas, Stud.I.C.E.
Ashworth, Robert, B.Eng. (*Liverpool*), Stud.I.C.E.
Aubrey, Kenneth George, Stud.I.C.E.
Baker, John Howard, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Baker, Peter, B.Sc. (*Birmingham*).
Bannister, Herbert David, B.Sc. (*Birmingham*), Stud.I.C.E.
Barbour, John Roy, Stud.I.C.E.
Barr, Alexander Fergus, B.Sc. (*Glasgow*), Stud.I.C.E.
Barsby, John Rossell, B.Sc. (*Belfast*).
Beattie, Alexander Ogilvie, B.Sc. (*Glasgow*), Stud.I.C.E.
Beavan, Gordon Cornelius Granville, B.Sc. (*Bristol*).
Beimel, Chaiym, B.Sc. (*Cape Town*).
Bell, Michael Norman, B.Sc. (*Birmingham*), Stud.I.C.E.
Bendall, David John Francis, Stud. I.C.E.
Bentley, Ivor, B.Sc. (*Manchester*), Stud.I.C.E.
Bickford, Norman, B.Sc. (*Wales*).
Bird, Alan Robert, Stud.I.C.E.
Birkett, Caryll Wreford, B.A. (*Oxon.*).
Blackham, Maurice Lafleur, Stud. I.C.E.
Blackledge, George Fort, B.Sc.Tech. (*Manchester*).
Bolland, John, B.Sc. (*Glasgow*).
Bond, John Gordon, B.Sc.Tech. (*Manchester*), Stud.I.C.E.
Bowman, Stephen Alexander Wallace, B.Sc. (*Bristol*), Stud.I.C.E.
Boxall, Lawrence Malcolm, B.Sc. (Eng.) (*London*).
Boydell, John Philip, B.Sc. (*Nottingham*), Stud.I.C.E.
Bracher, Geoffrey Herbert Winston, B.C.E. (*Melbourne*).
Braddock, Richard Walter, B.Sc. (Eng.) (*London*), Stud.I.C.E.
Branscombe, Charles John Stamp, B.Sc.(Eng.) (*London*).
Bregosz, Stanislaw, Stud.I.C.E.
Brennan, Robert Patrick, B.A., B.A.I. (*Dublin*).
Bridle, Ronald Jarman, B.Sc.(*Bristol*).
Bromley, Victor John, B.Sc. (*Cape Town*), Stud.I.C.E.
Brown, Frederick George, Stud.I.C.E.
Brown, James Kerr, B.Sc. (*Glasgow*).
Brown, Peter Rigby, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Browne, Maximillian George Burrowes, B.Sc. (*Belfast*).
Bryers, Thomas.
Buchanan, Neil, B.Sc. (*Durham*), Stud. I.C.E.
Burdett, John Fergusson, B.A. (*Cantab.*).
Burnett, Alan Belshaw, B.Sc. (*Belfast*).
Cadman, Kenneth John, B.Sc. (*Belfast*).
Cameron, Alexander Craig, B.Sc. (*Edinburgh*).
Canagarajah, Allen Chinnappah, B.Sc. (Eng.) (*London*).
Carey, Harry Neill, Ph.D., B.Sc. (*Durham*), Stud.I.C.E.
Carr, John Hallett, Stud.I.C.E.
Carson, James Humphreys, B.Sc. (*Belfast*), Stud.I.C.E.
Chan Nai Keong.
Chapman, David John, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Chard, Roy Creswell, B.Sc. (*Wales*), Stud.I.C.E.
Cheal, Brian David, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Chow David Slong Keng.
Christie, John, Stud.I.C.E.
Clark, Tony Allen, Stud.I.C.E.
Clark, William Michael, Stud.I.C.E.
Clarke, Michael Henry, Stud.I.C.E.

Clayton, Martin, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Cochrane, Stanley Robert, B.Sc. (*Belfast*), Stud.I.C.E.
Coffey, David Graham, B.A., B.A.I. (*Dublin*).
Cogan, Kenneth, Stud.I.C.E.
Collier, Douglas, Stud.I.C.E.
Coombs, David, B.Eng. (*Sheffield*).
Corby, Roderick Gordon, Stud.I.C.E.
Couper, James Thomas Lee, B.E. (*New Zealand*).
Craigie, Allan Sinclair.
Cranston, William James French, Stud.I.C.E.
Crawford, David Saxby, Stud.I.C.E.
Crockett, George Lumsden, B.Sc. (*St Andrews*).
Cronin, Denis Paschal, B.E. (*National*), Stud.I.C E.
Cross, Malcolm Stewart Barrington, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Crosthwaite, Donald Rothery, B.A. (*Cantab.*), Stud.I.C.E.
Crowley, Frank Walton, B.Sc. (*Leeds*), Stud.I.C.E.
Cunningham, Edward James, B.Sc. (*Glasgow*), Stud.I.C.E.
Cutler, Ronald Chandos, B.Sc. (*Birmingham*).
Dall, James Balfour, B.Sc. (*Glasgow*).
Davies, John Duncan, B.Sc.(Eng.) (*London*).
Davies, Peter William, Stud.I.C.E.
Davis, Peter Brian, B.A. (*Cantab.*), Stud.I.C.E.
Day, Alistair Scott, B.Sc. (*Bristol*).
Deane, Anthony Hubert, Stud.I.C.E.
Dent, John Arthur, Stud.I.C.E.
Dewhurst, Alan, Stud.I.C.E.
Dias-Abeyesinghe, Annesley Frederick, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Dickins, Kenneth William, Stud.I.C.E.
Dickson, John Fraser, Stud.I.C.E.
Dixon, Alan, B.Sc.(Eng.) (*London*).
Dixon, Richard Kennedy, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Dobbs, Robert Alan, B.Sc.(Eng.) (*London*).
Dockerty, Anthony, B.Sc. (*Durham*), Stud.I.C.E.
Donno, George Frederick, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Douglas, John George William, B.Sc. (Eng.) (*London*), Stud.I.C.E.
Douglass, Matthew McCartney, B. Eng. (*McGill*).
Edkins, Angus, Stud.I.C.E.
Edwards, Adam, Stud.I.C.E.
Edwards, John Michael, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Edwards, Keith William, Stud.I.C.E.
Edwards, Meirion, B.Sc. (*Bristol*), Stud.I.C.E.
El-Katib, Mohamed Tarik, Ph.D. (*London*), B.Sc. (*Fouad*), Stud.I.C.E.
Eriksen, Olav Malcolm Aunan, B.Sc. (*Wales*).
Ervine, Norman Derrick McKinlay, B.Sc. (*Belfast*), Stud.I.C.E.
Evans, David Wallace, M.A. (*Cantab.*), Stud.I.C.E.
Eveleigh, William James Geoffrey, Stud.I.C.E.
Fairweather, Chalmers Henry, Stud. I.C.E.
Farrell, Terence Hugh, B.Sc. (*Belfast*), Stud.I.C.E.
Faulkner, Derek Harding, B.A. (*Cantab.*), Stud.I.C.E.
Fernando, Harrison Samuel Gerald, B.Sc.(Eng.) (*London*).
Fieldhouse, Arnold Ernest, B.Sc. (*Bristol*).
Forge, Kenneth Robert Percy, Stud. I.C.E.
Fort, Geoffrey Buchanan, B.Sc. (*Manchester*), Stud.I.C.E.
Fretter, Peter John, Stud.I.C.E.
Fuge, Thomas Dymond, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Fuller, Raymond Thompson, B.Sc. (Eng.) (*London*).
Fulwood, Alan Frederick, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Gale, Ronald George, B.Sc.(Eng.) *London*), Stud.I.C.E.
Gallagher, Roy, Stud.I.C.E.
Garnham, John Henry, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Gaskell, William, B.A. (*Cantab.*).
Gates, John Colin, Stud.I.C.E.
Geatrell, Donald Ernest, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Genders, Paul Malcolm, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Gibson, George, B.Sc. (*Glasgow*), Stud. I.C.E.
Gilroy, John Morritt, B.Sc.(Eng.) (*London*).
Girdham, Ralphe, B.Sc.(Eng.) (*London*).
Gledhill, Geoffrey Maude, B.Sc. (Eng.) (*London*), Stud.I.C.E.
Glover, Alan, B.Sc.Tech. (*Manchester*).
Glover, Hubert Alan, B.Eng. (*Liverpool*), Stud.I.C.E.
Glover, John Benjamin, Stud.I.C.E.
Goodger, Donald, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Goonewardena, Reginald Stanley Fonseka, B.Sc.(Eng.) (*London*), Stud. I.C.E.
Graham, John Richard Neville, B.Sc. (*Belfast*).

Graham, Malise John, B.Sc. (*Cape Town*), Stud.I.C.E.
Grandfield, Donovan Hugh, Stud. I.C.E.
Green, Peter Albert, Stud.I.C.E.
Greenwood, Alan, Stud.I.C.E.
Gregory, John, Stud.I.C.E.
Gresswell, Peter John, B.A. (*Oxon*).
Gudgeon, Roger Wayth, B.E. (*New Zealand*).
Gunaratnam, Sinnappah, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Gunawardene, Gerard Vaz, B.Sc. (Eng.) (*London*).
Gunton, Robert Vanstone, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Gurowich, Peter, Stud.I.C.E.
Haines, Noel William, B.A. (*Cantab.*) Stud.I.C.E.
Hall, Charles Wilson, B.E. (*New Zealand*), Stud.I.C.E.
Hall, James Logan, Stud.I.C.E.
Hall, William Edwin, B.A. (*Cantab.*).
Hammett, Michael John Redwood, Stud.I.C.E.
Hamp, Eric James, Stud.I.C.E.
Hankinson, Harry, Stud.I.C.E.
Harker, Thomas Alan, B.Sc. (*Durham*), Stud.I.C.E.
Harris, Royston Sidney.
Hartland, Robert Arthur, Stud.I.C.E.
Hawkins, Theodore Francis John, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Haymes, Trevor Robert, B.Sc.Tech. (*Manchester*), Stud.I.C.E.
Hebron, Matthew Stephenson, B.Sc. (*Wales*).
Hedderwick, James Thomas, B.Sc. (Eng.) (*London*), Stud.I.C.E.
Henderson, Ronald Vernon, B.Sc. (*Belfast*), Stud.I.C.E.
Hicklenton, Brian Stuart, B.Sc. (*Leeds*), Stud.I.C.E.
Hill, Kenneth Edwin, Stud.I.C.E.
Hill, Martin Robert Ewbank, Stud. I.C.E.
Hind, John Ronald, Stud.I.C.E.
Hipkins, Michael Roland, B.E. (*New Zealand*), Stud.I.C.E.
Hirst, Peter Rushworth, B.Eng. (*Liverpool*), Stud.I.C.E.
Hitchins, Geoffrey Richard, B.Sc. (Eng.) (*London*).
Hodgkinson, Arthur, B.Sc.Tech. (*Manchester*).
Hopkins, John Price, B.Sc. (*Wales*), Stud.I.C.E.
Horner, Stuart Roy, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Horton, Kenneth Charles, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Howell, Albert Edgar, B.Sc. (*St Andrews*).
Howlett, Ronald William, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Huggins, Raymond Arthur, B.Sc. (Eng.) (*London*), Stud.I.C.E.
Hughes, Francis.
Hutton, George John, B.Sc.(Eng.) (*London*).
Hyde, John Edgar.
Illsley, Derek, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Ilse, Friedrich George, B.Sc. (*Cape Town*).
Ingoldby, Richard Cooper, B.A. (*Cantab.*).
Instance, Michael Courage, B.Sc. (Eng.) (*London*), Stud.I.C.E.
Irving, Derek Arthur, Stud.I.C.E.
Isaac, Peter John, B.Sc. (*Witwatersrand*).
Jack, Alexander, Stud.I.C.E.
Jackson, Anthony Derek Haygarth, B.A. (*Cantab.*).
Jayasooriya, Luddusinghabadu Samson De Silva, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Johnson, David Philip, B.A., B.A.I. (*Dublin*).
Jolly, John Harding, B.A. (*Cantab.*), Stud.I.C.E.
Jones, Henry James King, Stud.I.C.E.
Kay, Emrys, B.Sc. (*Wales*).
Kay, Terence, Stud.I.C.E.
Kelly, Rowland Edmund Osborne, B.Sc. (*Witwatersrand*).
Kennedy, Christopher David, Stud. I.C.E.
King, Eric, B.Sc. (*Manchester*).
King, John Robert James, B.Sc.(Eng.) (*London*), Stud.I.C.E.
King, Michael John, Stud.I.C.E.
Kinnear, Roy Graham, B.Sc. (*Glasgow*).
Kinsella, James, B.E. (*National*), Stud. I.C.E.
Kirk, Francis Charles Follett, B.Sc. (*Cape Town*).
Klein, Alan Harry, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Kumarasuriar, Chelliah, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Lancaster, Rex Ivan, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Latter, Bernard Victor, B.Sc.(Eng.) (*London*).
Lau Yat Sun, B.Sc.(Eng.) (*London*).
Lea, William Nigel, B.Sc. (*Birmingham*).
Lee, David Stanley, B.Sc. (*Wales*), Stud.I.C.E.
Leitch, Colin Walter, B.E. (*New Zealand*), Stud.I.C.E.

LINGWOOD, PETER NORMAN, B.Sc.(Eng.) (*London*), Stud.I.C.E.
LOGIE, JOHN GEORGE, B.Sc. (*Glasgow*), Stud.I.C.E.
LONG, WALTER BRIAN, B.Eng. (*Liverpool*), Stud.I.C.E.
LONGMUIR, HERBERT FULTON, Stud. I.C.E.
LOVEDAY, JOHN MICHAEL, B.Eng. (*Sheffield*), Stud.I.C.E.
LOWSON, WILLIAM WALLACE, B.Sc. (*Glasgow*), Stud.I.C.E.
LYNCH, BERNARD JOSEPH, B.Sc. (*St Andrews*).
MCAULEY, THOMAS JOHN ANDREW, B.Sc. (*Belfast*), Stud.I.C.E.
MCCALLUM, DAVID BRADDON, B.Sc. (Eng.) (*London*), Stud.I.C.E.
MCFADZEAN, MATTHEW, B.Sc. (*Glasgow*), Stud.I.C.E.
MCKAY, COLIN, Stud.I.C.E.
MCKENZIE, THOMAS MELVILLE BINGHAM, Stud.I.C.E.
MCKIBBIN, THEODORE GEORGE HENRY, B.Sc. (*Belfast*).
MCLAREN, RICHARD DONALD, B.E. (*New Zealand*), Stud.I.C.E.
MACLENNAN, WILLIAM GREGOR, B.Sc. (*Leeds*), Stud.I.C.E.
MAHALINGAM, SELVADURAI, B.Sc.(Eng.) (*London*), Stud.I.C.E.
MAISEL, BRUCE ISAAC, B.Sc. (*Witwatersrand*).
MARAIS, CLAUDE PIERRE, B.Sc. (*Natal*).
MARK, EDWIN, B.Sc.Tech. (*Manchester*), Stud.I.C.E.
MASON, JAMES ROY, Stud.I.C.E.
MAY, JAMES ROBERTSON, B.Sc. (*Aberdeen*).
MAYNARD, CEDRIC WALTER, B.Sc.Tech. (*Manchester*), Stud.I.C.E.
MEGIT, JOHN SYDNEY CHARLES, B.Sc. (Eng.) (*London*), Stud.I.C.E.
MELDRUM, DOUGLAS, B.Sc. (*Glasgow*).
MILLER, DAVID YOUNG, Stud.I.C.E.
MILLER, PETER TEMPLE STEUART, B.Sc. (*Cape Town*).
MINCH, DAVID BERCHMANS, B.A., B.A.I. (*Dublin*), Stud.I.C.E.
MITCHELL, JOHN CORY, B.Sc. (*Birmingham*).
MONCRIEFF, DAVID GEORGE, B.Sc. (*St Andrews*).
MOORHOUSE, EDWARD, B.Sc.(Eng.) (*London*).
MORGAN, DONALD, B.Sc. (*Wales*), Stud. I.C.E.
MORNEMENT, PETER CLEWS, B.A. (*Cantab.*), Stud.I.C.E.
MORRIS, PHILIP HENRY, B.Sc.(Eng.) (*London*).
MORRISS, ROLAND HENRY, Stud.I.C.E.
MOULT, ANTHONY VINCENT, B.Eng. (*Sheffield*), Stud.I.C.E.
MOULTON, GERALD DOUGLAS, Stud.I.C.E.
MOWATT, ALBERT ERNEST, B.Sc. (*Glasgow*), Stud.I.C.E.
MUNRO, ANGUS TOD, Stud.I.C.E.
MYNOTT, PETER JACK.
NAUGHTON, AIDEN JOSEPH, B.E. (*National*), Stud.I.C.E.
NEIL, DOUGLAS GRANT, B.Sc. (*Glasgow*), Stud.I.C.E.
NEWELL, DAVID HALL, Stud.I.C.E.
NEWING, JOHN LEWIS, B.Sc. (*Bristol*), Stud.I.C.E.
NICOL, HUGH BELL ROLLO, Stud.I.C.E.
NISH, IAN CRAWFORD, B.Sc. (*Edinburgh*), Stud.I.C.E.
NIXON, JOHN BRIAN, B.Sc.(Eng.) (*London*), Stud.I.C.E.
NORMAN, JOHN LEONARD.
NORTH, JOHN DURHAM, B.A. (*Cantab.*), Stud.I.C.E.
OLIPHANT, EBENEZER, Stud.I.C.E.
OLIVIER, PAUL ANTHONY, B.Sc. (*Cape Town*).
O'RORKE, GEORGE BRYAN, B.A. (*Cantab.*).
PALMER, MICHAEL ALAN, B.Eng. (*Sheffield*).
PARKER, WILFRED HARRY, B.Sc. (*Durham*), Stud.I.C.E.
PARKIN, LESLIE WILLIAM.
PARSLOW, FRANK DOMINIC, B.Sc. (*Belfast*), Stud.I.C.E.
PATTISON, JAMES, B.Sc. (*St Andrews*).
PEARCE, ROGER LE BLOND, B.Sc.(Eng.) (*London*), Stud.I.C.E.
PEDDER, WILLIAM MORRIS, Stud.I.C.E.
PENNINGTON, JOHN STANLEY, B.A. (*Cantab.*).
PERFREMENT, DENIS, B.Sc. (*Durham*), Stud.I.C.E.
PINK, VICTOR ALAN, B.Sc.Tech. (*Manchester*), Stud.I.C.E.
PITTS, JOHN BLEASDALE, Stud.I.C.E.
POLLIN, JOHN, Stud.I.C.E.
POWELL, RALPH MORRIS, Stud.I.C.E.
POWELL, THOMAS GERALD MICHAEL GLYN, B.Sc.(Eng.) (*London*), Stud. I.C.E.
PRATT, GEOFFREY CYRIL, B.Sc.(Eng.) (*London*), Stud.I.C.E.
PRESTON, PETER FREDERICK BROWNING, Stud.I.C.E.
PRIOR, GEOFFREY, B.Sc.Tech. (*Manchester*), Stud.I.C.E.
PUN YIN KEUNG, Stud.I.C.E.
PYRAH, DAVID ERIC, Stud.I.C.E.
RACE, DENYS ARTHUR, B.Sc.(Eng.) (*London*), Stud.I.C.E.
RADFORD, JAMES, B.Sc. (*Belfast*), Stud. I.C.E.

Raine, James Brailsford, B.A. (*Cantab.*), Stud.I.C.E.
Ratnarajah, Ackland Rasiah, B.Sc. (Eng.) (*London*), Stud.I.C.E.
Ravine, Robert Peter, Stud.I.C.E.
Ray, Dennis Keith, B.Sc.(Eng.) (*London*).
Reay, Geoffrey Cecil, B.Sc. (*Durham*), Stud.I.C.E.
Rees, Malcolm Randall, B.Sc. (*Wales*), Stud.I.C.E.
Richards, Geoffrey Julian Miller, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Richards, Gwillim Thomas, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Rigby, Arthur David, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Robertson, James Orr, B.Sc. (*Glasgow*), Stud.I.C.E.
Robinson, George Edward, B.Sc. (*Belfast*), Stud.I.C.E.
Robinson, Thomas Leonard, Stud.I.C.E.
Roe, Michael William, B.Eng. (*Liverpool*), Stud.I.C.E.
Sanders, Donald Reginald.
Saner, John Eric David, Stud.I.C.E.
Saravanapavan, Arumukham.
Sargent, Derek Edward William, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Saunders, Geoffrey Charles, B.Sc. (Eng.) (*London*), Stud.I.C.E.
Schafer, David William, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Scott, David Peel.
Scott, James Grant, B.E. (*New Zealand*).
Scowsill, Peter John, B.A. (*Cantab.*).
Senaratne, Makewitage Johannes Francis Raphael, B.Sc.(Eng.) (*London*).
Senior, Edward, Stud.I.C.E.
Shardlow, Ambrose John, B.A. (*Cantab.*), Stud.I.C.E.
Sharpe, Ian David, M.A. (*Cantab.*), Stud.I.C.E.
Shawcross, John Fawley, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Shipway, James Simpson, B.Sc. (*Glasgow*), Stud.I.C.E.
Shores, Kenwyn Douglas Tatham, B.E. (*New Zealand*).
Short, Roger Shaun, Stud.I.C.E.
Siddall, James Gordon, Stud.I.C.E.
Simpkin, Gordon, Stud.I.C.E.
Singh, Sandhu Pritam, B.Sc. (*Glasgow*).
Small, David Ian, B.Sc. (*Edinburgh*), Stud.I.C.E.
Smallman, Anthony Joseph, B.Sc. (Eng.) (*London*), Stud.I.C.E.
Smith, Derek Reginald, Stud.I.C.E.
Smith, Iain Parker, B.Sc. (*St Andrews*).
Smith, John Brian, B.Eng. (*Liverpool*), Stud.I.C.E.
Smith, Sidney Henry John, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Smith, William Thomas, B.Eng. (*Liverpool*).
Sparrow, Robert Watson, B.A., B.A.I. (*Dublin*).
Spiers, Brian Clifford, B.Sc. (*Cape Town*).
Stables, Frank Peter David, Stud. I.C.E.
Stafford, Keith Charles Stapleton, Stud.I.C.E.
Strickland, Ronald MacKenzie, B.E. (*W. Australia*).
Sunderland, Patrick Hugh, Stud. I.C.E.
Sunnak, Sardari Lal, B.Sc.(Eng.) (*London*).
Sutcliffe, Frank.
Sutton, Reginald James.
Swabey, Norman Albert Ormiston, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Tasker, Alexander Macdonald, B.Sc. (*Durham*), Stud.I.C.E.
Tasker, John Antony Leonard, B.Sc. (Eng.) (*London*), Stud.I.C.E.
Tate, Adrian Peter Kisse, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Taylor, Howard Royce, B.Sc.Tech. (*Manchester*), Stud.I.C.E.
Taylor, Robert Brian, Stud.I.C.E.
Teller, Otto George, Stud.I.C.E.
Thackeray, Adam, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Thirst, Alan Fletcher, Stud.I.C.E.
Thomas, David, B.Sc.Tech. (*Manchester*), Stud.I.C.E.
Thompson, Norman Everatt, B.Sc. (*Nottingham*), Stud.I.C.E.
Tokarski, Eugeniusz Wladyslaw, Stud.I.C.E.
Tomasin, Kenneth, Stud.I.C.E.
Townson, Arthur, B.Sc.Tech. (*Manchester*), Stud.I.C.E.
Truebody, Charles Thomas, B.Sc. (*Cape Town*), Stud.I.C.E.
Tucker, William Duncan Alistair, B.A. (*Oxon.*), Stud.I.C.E.
Tunnell, Thomas Edmund, Stud.I.C.E.
Tunstall, Alan, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Turner, Geoffrey, Stud.I.C.E.
Turner, Henry George, Stud.I.C.E.
Underwood, Brian Leslie, Stud.I.C.E.
Ure, Alan Willis, B.A. (*Cantab.*), Stud.I.C.E.
Valpy, Douglas Burnside, B.Sc. (*Natal*).
Vaughan, Alan Arthur Philip, Stud. I.C.E.
Venter, Jacobus Johannes, B.Sc. (*Witwatersrand*).

Viapree, Ronald Joseph Kerr, B.Sc. (*Edinburgh*), Stud.I.C.E.
Walker, Adam Fulton, Stud.I.C.E.
Walsh, Thomas, B.Eng. (*Liverpool*).
Walsh, Kenneth James, B.Sc.Tech. (*Manchester*), Stud.I.C.E.
Wardle, John Barber, B.Sc. (*Birmingham*).
Watkinson, John Stuart, B.Sc. (*Leeds*), Stud.I.C.E.
Watson, Derrick, Stud.I.C.E.
Watson, James Alexander Stevenson, B.Sc. (*Edinburgh*), Stud.I.C.E.
Watts, Derek John Bryant, B.Sc. (Eng.) (*London*), Stud.I.C.E.
Webster, William Duncan, B.Sc. (*St Andrews*).
Weir, William, B.Sc. (*Glasgow*), Stud. I.C.E.
Wheater, John Denis Robert, Stud. I.C.E.
White, Graham Charles Sidney, B.E. (*New Zealand*), Stud.I.C.E.
Whitehouse, John Frederick, B.Sc. (Eng.) (*London*), Stud.I.C.E.
Wilkinson, John Michael, B.Sc. (*Leeds*), Stud.I.C.E.
Willey, Edward Walton, B.Sc. (*Birmingham*).
Williams, Francis Clive, B.E., (*New Zealand*), Stud.I.C.E.
Williams, William George Anthony, B.Sc. (*Wales*), Stud.I.C.E.
Williamson, Frederick Jack, B.Sc. (Eng.) (*London*), Stud.I.C.E.
Willington, Thomas William, M.Sc. (*Belfast*).
Wilson, John Fleming, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Wilson, John Spedding, B.Sc.Tech. (*Manchester*).
Wilson, Thomas Gilmour, B.Sc. (*Belfast*), Stud.I.C.E.
Winfield, Alan Frederick, B.Sc.(Eng.) (*London*).
Winstone, Anthony Ronald, Stud. I.C.E.
Wishart, John Thomas Alston, B.Sc. (*Edinburgh*), Stud.I.C.E.
Wood, Donald Frederick, B.Sc. (*Durham*), Stud.I.C.E.
Woolston, Geoffrey Haig, B.Sc.(Eng.) (*London*), Stud.I.C.E.
Wragg, Frank Anthony, Stud.I.C.E.
Wright, Kenneth, Stud.I.C.E.
Wright, Lewis Hamilton, B.Sc. (*Glasgow*), Stud.I.C.E.
Wright, Michael Stephen, B.Eng. (*Sheffield*), Stud.I.C.E.
Wroe, Donald Arthur, B.Sc.Tech. (*Manchester*), Stud.I.C.E.
Wynne-Jones, Randell Devereux, B.Sc.(Eng.) (*London*).
Yeo, Peter Douglas, B.Sc. (*Cape Town*).
Zahorski, Witold, B.Sc.(Eng.) (*London*), Stud.I.C.E.

and had admitted as

Students

Affleck, Gerald Gordon.
Aird, Robert Malcolm.
Alcock, John.
Allan, David Alexander.
Allen, Ronald.
Austin, Peter Benjamin Magnus.
Bagshaw, Grenville.
Banks, Peter Alan.
Barnbrook, Malcolm Grove.
Barratt, Gordon.
Barratt, Michael John.
Bass, David William, B.Eng. (*Liverpool*).
Bateman, Ivor Henry.
Bedford, Michael John.
Bowers, Michael Geoffrey.
Brazier, Murray William Weth.
Brierley, John Stewart.
Burnell, Ian Derek.
Carter, Brian Mackfall.
Carter, Howard.
Cavin, Leonard.
Challenor, John Haden.
Charles, Avondale Morris.
Cheong Gum Chu.
Chung, Jerome Albert Wilkinson.
Cole, Francis Arthur.
Corsie, Olaf John.
Court, Colin.
Cowburn, Michael John.
Crook, Peter Benjamin John.
Davey, Robin Trelace.
David, Joseph Nessim.
Davies, John Edmund.
Davis, Robert Oswald.
Deans, Kenneth John.
Dixon, John George.
Donkin, Alfred Walter.
Drabble, Peter Gordon.
Duncanson, John King.
Edwards, Kenneth Peter.
Elford, Victor Henry.

Entecott, Edwin Charles.
Foord, Michael John.
Forsyth, Donald Hugh Darg.
Foulkes, Derek Joh.
Fox, John Arthur.
Freckingham, John Marcus.
Ganesan, Mahesan.
Gibbon, Alan.
Giles, Harry Joseph.
Gill, Michael Brendan Mary.
Gillis, Lawrence.
Goldsmith, Paul.
Goodwin, John Albert.
Green, John David.
Grierson, Donald.
Gunewardena, Wanusapperumaarachige Darwin.
Gwynn, Derek Ronald.
Hargreaves, Derek James.
Harold-Barry, Harold Robin.
Harper, Raymond Stubbs.
Harrison, Glen Allen.
Hasson, Raymond Rachamin Hasda.
Hayes, Anthony Edward John.
Head, Gordon Richard Arthur.
Hepburn, James Cochrane Joseph.
Herbert, Dion Curtis James.
Hesketh, Alfred Glyn.
Hill, Gordon.
Hill, Sidney David.
Hillcox, John Barry.
Hillis, Sydney Fraser.
Hobbs, Derek.
Holding, Neville William.
Horn, David Thomas.
Hulme, John.
Hunt, George Reginald Yerusatnam.
Hutchinson, William Henry Barry.
Hyde, Thomas Michael.
Hyland, Malcolm Peter.
Iliffe, Frank Joseph.
Iliffe, Richard.
Ironside, Lester Wilson.
Jackson, Peter Alistair.
Jones, John Clifford.
Jones, Royston William.
Johnson, Michael Alan.
Kanagalingam, Candiah.
Karasek, Colin Joseph.
Kilborn, Neville Stanley.
Kille, Vivian Edwy.
Kliegl, Walter John David.
Krumm, Michael George.
Lack, William Brian.
Laird, Robert Alexander.
Lane, Bryan Neil.
Lane, Peter John Ford.
Law, Keith Edward
Lawson-Williams, Ian Malcolm.
Lees, Ivan Johnson.
Lenihan, Peter William.
Lewis, Brian William.
Lewis, Ronald Thomas.
Light, Robert William Arthur.
Ling, David Legrys.
Logan, Samuel Forbes.
Lyon, Rodney Leroy.
McColgan, Neil.
McCormack, Daniel Thomas.
McCracken, David Mitchell.
McDougall, Dougal.
McEwan, David Murdoch.
MacKechnie, William Ronald.
Mackie, Andrew Duncan.
Malcolm, James Graham.
Maniarpillay, Velluppillai Kailasapillai.
Marshall, Clive Turner.
Marzook, Mohomed Shariff Mohomed.
Masojada, Miloslaw Edmund.
Maylan, Arthur Geoffrey.
Middleton, Ronald Gustav.
Mitchell, Walter Tolton.
Murray, Elliot Lionel.
Nanda, Vidya Sagar Roora Mal.
Nicholson, Donald McGregor.
Nutt, Richard David.
O'Donovan, Diarmuid Columcille.
Oglesby, Keith.
Packer, Geoffrey Albert.
Pancholi, Vijayshanker Ravishanker.
Panditaratne, Kanakahewage Sisara.
Paramadas, Joseph.
Park, George Thomson Booth.
Pemberton, Colin Stanley.
Perera, Ivan Hilarian.
Porter, Allan Ernest.
Price, Anthony David.
Priest, David Edward.
Prior, Anthony James.
Prior, Peter Donald.
Rawlings, Trevor John.
Reilly, Francis.
Revell, Lionel Robert.
Roberts, Anthony Charles Pierce.
Robertson, Ian Russell Struan, B.Sc. (*Hong Kong*).
Robinson, Frank.
Roddan, Homer.
Rumble, James Edward.
Russell, Kenneth Sidney.
Rutherford, Henry.
Ruxton, James Archibald.
Rynkiewicz, Leonard Stanislaw.
Sawyer, John Farrington.
Scanlen, Eric Alexander.
Scone, Derek Vivian.
Sethi, Satindra Nath.
Sion, Edward Robin.
Smalley, Lawrence Stone.
Smith, Alan.
Smith, Frank.
Soulsby, Philip John.
Spence, Malcolm William.

Springett, Peter Roy.
Squires, Colin John.
Stevens, Anthony.
Stockdale, John Kenneth.
Stocks, Robert.
Stone, William John Kingston.
Sullivan, Richard Arthur.
Tennekoon, Kenneth Dudley.
Thistlethwaite, Gerard.
Thöle, Lyle Fleming.
Thomas, Anthony Ronald.
Thomas, Elwyn Owen.
Thompson, Thomas Walter.
Thorn, Christopher Hammond.
Tindall, Leonard Charles.
Tinkler, Basil Ashton.
Todd, James Richard.
Tomlinson, Edward.
Tracey, Alfred Charles.
Turner, Charles Peter Owen.
Twigg, David Joseph.
Udall, Robert Denton.
Veeraraghavachary, Krishnamachary.
Vickerton, Peter Allan.
Waddington, Kenneth.
Wainwright, Alexander Casely.
Walsh, Gabriel Joseph Patrick.
Waring, Robin Bryan.
Waters, Brian Edward.
Watkeys, Douglas Gwyn.
Watkins, David John.
Weaver, John Frederick.
Welshman, Terence George Frederick.
West, Christopher Leon.
West, David Robert.
Westermann, Reginald Charles.
White, Malcolm Francis George.
Whitting, Alan Charles.
Wilkie-Snow, George.
Wilson, Michael Stuart.
Withall, William Nigel James.
Woldeabriel, Beyene.
Wong Mang-Ki, B.Sc. (*Hong Kong*).
Wood, Martin Hemsley.
Woodall, Robert Timothy John.
Wright, Lewis Hamilton.

The Secretary announced the awards which had been made by the Council for Papers read at meetings in Session 1952–53. Full details of these awards are given on p. 43.

A number of the recipients were in attendance and the President made the following presentations:

Coopers Hill War Memorial Medals and Certificates to Mr R. S. Jenkins, who received them on behalf of Mr O. N. Arup and himself.

Certificates of the award of Telford Premiums to Messrs H. D. Morgan, P. A. Scott, R. J. C. Walton, R. H. Falkiner, D. H. New, J. A. Derrington, A. G. S. Lance, Dr E. Windle Taylor, Messrs D. J. Maclean, P. J. M. Robinson, and P. O. Wolf.

Certificates of the award of a Trevithick Premium to Mr R. E. Sadler and of a Crampton Prize to Mr P. A. Scott.

An Institution Medal to Mr J. G. Measor.

Messrs E. S. Crump, Z. S. Makowski, and F. J. Samuely were also present and received Certificates of the award of Telford Premiums for Papers published with written discussion in the year 1952.

The Retiring President, introducing the new President to the meeting, said that Mr Shepherd-Barron had just retired from the position of Chief Engineer of the Port of London Authority; his work in the past six or seven years had been one of unremitting toil to bring the Port of London back into its pre-war state. Two of his predecessors had already held the office of President, and there was no doubt that he would prove a very worthy successor to them.

Mr Cronin knew that members would extend to Mr Shepherd-Barron the kindness and courtesy which he had himself received ; and in inviting the new President to take the Chair he wished him a very happy and successful year of office.

Mr Wilfrid Philip Shepherd-Barron, M.C., T.D., then took the Chair as President and called on Mr W. K. Wallace to move a resolution.

Mr W. K. Wallace, Vice-President, moved the following resolution :

" That the members present at this Meeting desire, on behalf of themselves and others, to record their high appreciation of the services rendered to the Institution by Mr Cronin during his term of office as President."

Although the resolution, if put formally to the meeting, would no doubt be passed unanimously, he felt that he would be wanting in courtesy to the retiring President if he did not give some idea of the very high regard of the members of the Council who had worked with him during the year. Mr Cronin was a regular attender at meetings and controlled the Council remarkably well—not that they were normally a disorderly body, but he got business through very pleasantly and promptly.

It had been a fairly normal year. The most outstanding event, from the Institution's point of view, had been the extensive flooding on the East Coast early in 1953, when Mr Cronin had taken prompt steps to get together an expert Committee of the Institution in order to produce a memorandum to help the " Waverley " Committee in their deliberations on what had occurred and how to prevent its being a serious risk in the future. Later, when that Committee had asked the President to appoint members to give evidence before them, Mr Cronin had made a very wise selection of gentlemen who had been of greater assistance to that Committee.

It might be suggested that the Institution was at present in what would be called by astrologers the region of the House of Aquarius. They had just had a President who had been interested in clean water ; they were going to have a President who was interested in salt or, possibly, brackish water ; and after that, if all went well, they would have a President who was interested in most deplorable water. If they succeeded as well as had Mr Cronin, and weathered the storms and any rough weather in the same way as he had done, the Institution would be very well satisfied.

In wishing Mr Cronin a very happy time as Past-President, Mr Wallace said that he had appeared to have enjoyed his Presidency very much, and he certainly had made it very pleasant for the Institution. Amongst his many natural advantages was a very pleasant manner. He could sometimes get his own way without others being aware of it. Perhaps that was partly because he was an Irishman in England.

Mr J. A. Banks, who seconded the motion, echoed Mr Wallace's remark that it was perhaps Mr Cronin's country of origin which had made

him such an approachable individual, even to the most junior member of the Institution. Whether in Committee or Council or *ex cathedra*. he had conducted affairs with a full understanding of the opinions and views of others.

Mr Cronin's initiative had always been tempered by a complete realization of the need to preserve the dignity of the Institution, and this he had done at all times. He had had a strenuous year of office, which would be memorable to him if only because it had been Coronation year. He could retire with the feeling that his name stood on the records of the Institution as one who had upheld its standing and dignity and who had been a worthy successor to all those who had preceded him.

The motion was carried with acclamation.

Mr H. F. Cronin, in acknowledging the vote of thanks, said that although it was a time-honoured custom to propose a vote of thanks to the retiring President, he felt it should be the other way round and the retiring President should propose a vote of thanks to the members and to the Council for electing him their President. They had done him a great honour, he said, and he was grateful for the way in which the resolution had been received.

The President then delivered the following Address.

PRESIDENTIAL ADDRESS OF

Wilfrid Philip Shepherd-Barron, M.C., T.D.
President, 1953–54

Introduction

To become the President of the Institution of Civil Engineers is the highest honour in our profession and I desire at once to say how much I appreciate the distinction of your electing me to hold this office. Although the prospect of the responsibilities which lie ahead gives rise to some misgivings, I can assure you that I shall do what I can to serve the Institution and the profession.

I have taken as the subject of my Address " The Docks of London," which together form the largest area of enclosed dock space within the United Kingdom and which are now owned and managed by the Port of London Authority.

These docks have a long history from the time of their several constructions, beginning a century and a half ago and extending during their development and improvement down to the present day, when further large and important engineering works of this nature are in progress or projected. Several of our former Presidents have been responsible for the construction of these dock systems and various Papers have been presented to the Institution from time to time describing these important works. It is my purpose to trace briefly the evolution of the docks as a whole and their engineering features which have changed in accordance with the circumstances of the times.

All the docks, with the exception of King George V Dock, originated as the properties of various dock companies, which from time to time were amalgamated and finally were taken over in 1909 by the Port of London Authority constituted by Act of Parliament.

There are five groups of docks (see Figs 1, Plate 1), namely :—

1. London and St Katharine Docks.
2. Surrey Commercial Docks.
3. East and West India and Millwall Docks.
4. Royal Victoria, Royal Albert, and King George V Docks.
5. Tilbury Docks.

The principal features and dimensions of these docks are given in Table 1.

Maritime trade has been conducted on the River Thames and in the Port of London since early times, and for many centuries the small vessels

engaged therein used to berth at wharfs in the Pool of London or at small tidal drawdocks excavated in the sides of the river, where they took the ground at low water. Later, as vessels became larger, they moored in the stream and were discharged and loaded by means of lighters of from 20 to 70 tons burthen. The tidal range in the Thames at London is considerable, being about 21 feet at springs, and gradually the idea of enclosed wet docks, in which vessels could lie afloat in safety and with greater convenience, became recognized.

From the time of the Middle Ages to the eighteenth century a number of small dry docks had been built on the banks of the river. These have disappeared but have been superseded by several larger ones.

Pepys recorded in his Diary, under the date 15th January, 1661 :—

> " So after a cupp of burnt wine at the taverne there [Deptford], we took barge and went to Blackwall, and viewed the dock, and the new West Dock, which is newly made there, and a brave new merchantman which is to be launched shortly, and they say to be called the Royal Oake."

The dock he referred to was a small fitting-out basin of $1\frac{1}{2}$ acre with gates to retain the water, and appears to be the earliest wet dock that was built.

In 1703, a wet dock had been built by private owners at Rotherhithe, known as the Howland Great Wet Dock, which was about 12 acres in extent. It was entered by means of a lock, 150 feet long by 44 feet wide, with a depth of 17 feet at H.W.O.S.T., and was large enough to admit the largest merchant ships of that date. It was used, however, merely for the shelter and repair of ships and not for their discharge —mainly because of difficulties with Customs—and so was not provided with sheds or warehouses. Its walls were made of timber campshedding. In 1763 it was sold, reconstructed by a Mr Wells, a local shipbuilder, re-named Greenland Dock, and used for the whaling trade ; in 1806 it changed hands again and was used for the timber trade, the name being changed to Commercial Dock.

In 1789, Mr Perry, the famous shipbuilder, constructed at Blackwall for the East India Company, a wet dock, consisting of two basins each of 8 acres, with entrances and a communicating lock. It was known as the Brunswick Dock and was used for the repair and fitting-out of the Company's vessels. It was provided with a large mast house, about 120 feet high, for stepping masts.

The mercantile shipping and traffic in the port expanded greatly during the eighteenth century and reached such proportions that there was very serious congestion in the river, causing great inconvenience and delay in landing and loading merchandise, which, in turn, assisted the growth of intolerable theft and organized pilferage of all kinds ; the loss to traders was very heavy.

In 1700, the total value of imports into the port was £4,875,538 and of exports £5,387,787, and by 1798 these figures had increased to £13,065,290 and £18,002,204 respectively. In 1700, ships averaged about 100 tons burthen and, in 1800, about 200 tons.

The development of the West Indian colonies had led to a large increase in the importation of dutiable goods, chiefly sugar and rum, and these with all other dutiable goods such as tobacco, wines, and spirits could be landed from lighter only at the Legal Quays immediately below London Bridge on the north bank and at certain other quays under licence, mainly on the south bank, known as the Sufferance Quays. (The Legal Quays, built in the fifteenth century, were 1,500 feet long, and the Sufferance Quays were 3,700 feet long; no other expansion of accommodation had taken place.) This led finally to the formation of a Committee of Merchants under the leadership of William Vaughan, who petitioned Parliament with a view to the construction of enclosed wet docks. Various schemes were put forward by this Committee, the West India merchants, the City Corporation, and independent persons.

Eventually an Act of Parliament was passed in 1799 for the construction of the West India Docks in the Isle of Dogs by the West India merchants. These docks were completed in 1802 and the City Canal, south of them across the Isle of Dogs, was opened by the City Corporation in 1805. The latter was provided with locks at each end and was intended to obviate the long passage round the U-bend in the river; it was, however, a failure and was sold in 1829 to the West India Docks Company and later enlarged into a wet dock.

Before tracing the construction of the various dock systems it is necessary to mention certain natural features which relate to or have influenced the works.

Tidal Levels

A datum of levels was established at the Hermitage entrance of the London Docks, which corresponded with a mark established in 1800 by Captain Huddart, known as Trinity High Water and supposed to be the level of H.W.O.S.T., although in fact it is somewhat below that level at that place. Various other marks have been established from time to time at other places purporting to be at the same level but, owing to errors in levelling and settlement of structures and probably of the ground, discrepancies of a few inches have arisen. The original mark was 12·53 feet above the Liverpool Ordnance Datum and Trinity High Water was usually regarded as being 12·50 feet above that datum; it has now been established by statute as 11·40 feet above Newlyn Datum (11·10 feet at Southend).

Table 2 contains the principal tidal features of the river, the levels being related to Newlyn Datum and Trinity High Water.

Geology

The Thames valley through and below London lies above an extensive depression in the chalk, which is overlain by various later strata, namely, the Reading and Woolwich beds of sand, clay, and gravel, the Thanet sand, the London clay, gravel known as Thames ballast, and mud or soft marsh clay interspersed with layers of peat.

It has been fortunate that the Thames ballast stratum extends almost everywhere beneath the dock areas and at convenient levels at each of them, so that in the main it has been possible to build the dock walls, lock entrances, and other foundations upon it. The material in the undisturbed state is very dense, with a weight in air of about 116 lb. per cubic foot and an angle of internal friction up to about 42 degrees. It is very well graded and is a splendid natural aggregate for mass concrete, for which it has been used in the dock constructions.

At the London and St Katharine Docks the gravel lies at 9 feet below N.D. and is from 13 feet to 17 feet thick lying over the London clay.

At the Surrey Commercial Docks and Millwall Docks the London clay is almost absent. The gravel lies over the Reading and Woolwich and Thanet beds and is more variable in depth and thickness ; in some places foundations have had to be made on the latter strata with somewhat greater difficulty.

At the West India and East India Docks the gravel lies at from 3 feet to 16 feet below N.D. and is from 14 feet to 24 feet thick, lying over the London clay.

At the Royal Docks the gravel lies at from 7 feet at the western end to 20 feet at the eastern end below N.D. and is about 20 feet thick. At the west end of this system it lies over the London clay but towards the eastern end the clay disappears and the gravel lies over the chalk. The chalk outcrops at Purfleet, at Gravesend, and in the hills to the north of Tilbury ; between the last two places it has been scoured to form a submerged valley at a level of about 55 feet below N.D. overlain by gravel of thickness ranging from 8 feet to 30 feet, and then by very soft mud-clay and layers of peat.

West India Docks

The West India Docks were promoted by a company of West India merchants and were constructed between 1799 and 1802 on open marsh land in the Isle of Dogs, Mr W. Jessop and later Mr (later Sir) John Rennie, F.R.S., Past-President I.C.E., being the Engineers. They were a magnificent conception for the time and, with certain developments, have been in service ever since.

There were two docks, the Import and Export Docks, each 2,600 feet

long, the former 500 feet wide and the latter 400 feet, the depth being 24 feet at H.W.O.S.T. There was an oval-shaped basin at the eastern end of 6¾ acres extent, known as the Blackwall Basin, and a smaller one of 1¼ acres at the western end, known as the Limehouse Basin. Each of these basins communicated with both docks and with the river by locks; shipping used the eastern basin and lighters the western. The largest ship of the time in the West Indian trade was of 350 tons burthen but the river lock, which was 200 feet long, 45 feet wide, and 21 feet deep at H.W.O.S.T., could accommodate a ship of 1,200 tons burthen. The other locks were somewhat smaller.

The dock walls were of curved shape in section, provided with counterforts at the back, and were founded on the gravel. The shape was no doubt derived from the general idea of conforming to the natural concave shape of a clay bank and providing a revetment, rather than from any analysis of forces and moments; nevertheless the walls have stood satisfactorily, owing, no doubt, to the excellent foundation. A splendid range of five-storeyed timber-floored brick warehouses was built on the north side of the Import Dock. They were largely destroyed by fire during the 1939–45 war. The quay was provided with a narrow shed and later with hydraulic cranes.

Similar but lower warehouses were built on the tongue of land between the docks; some of them had vaults for the storage of wines and rum, and the Export Dock had a number of sheds.

In 1866, the Company, which had earlier acquired the City Canal to the south of their docks, enlarged it to form the South Dock. Mr (later Sir) John Hawkshaw, F.R.S., Past-President I.C.E., was the Engineer, L. F. Vernon-Harcourt being the Resident Engineer. The dock was 2,650 feet long by 450 feet wide and at the western end the old canal lock was retained. At the eastern end a basin was built, 600 feet long by 370 feet wide; a passage, 55 feet wide and 27 feet deep below T.H.W., between the dock and basin was fitted with two pairs of reverse gates. The new river lock was 300 feet long by 55 feet wide by 27 feet deep below T.H.W. (see Figs 3, Plate 2). The dock was 29 feet deep below T.H.W. and the bottom was covered with 2 feet of puddle clay over the gravel.

The dock walls were of rectangular section (Figs 2, Plate 2) on a lime-concrete base, and were built of brickwork with hollow cells filled with lime concrete. The lock walls were of similar construction, the invert being a brickwork arch on a lime-concrete base. The sills and quoins were of granite and double-skin wrought-iron gates were provided.

London Docks

The London Docks were built in stages by a separate company of merchants on ground at Wapping and Shadwell and involved the clearance

of much primitive dwelling property. Rennie and Alexander were the Engineers. The first stage, built between 1802 and 1805, consisted of the Western Dock and Wapping Basin with a lock between them, and the Wapping entrance-lock.

The Western Dock is rectangular in shape, 1,260 feet long by 690 feet wide, and the basin is about 3 acres in extent; both are 20 feet deep below T.H.W. The walls—of similar type to those at West India Docks—are of brickwork 6 feet thick with counterforts 15 feet apart (see Figs 2, Plate 2). They were founded on beech and elm planking direct on the gravel, with timber sheet-piling under the toes. The locks are 170 feet long by 40 feet wide by 23 feet deep below T.H.W. and are built of masonry with curved walls and a curved masonry invert laid on a brickwork base. The gates were of timber and wrought-iron composite construction with cast-iron sluices. The locks are crossed by double-leaf swivel bridges with cast-iron ribs.

It is interesting to note that when making the coffer-dam in the river at the lock, an 8-horse-power steam winch was used for driving the timber sheet-piles—the first recorded instance of steam being used for this purpose. The pumps were driven by a 25-horse-power Boulton and Watt steam engine.

The dock was surrounded by four-storey brickwork warehouses with timber floors having trussed beams and cast-iron columns; many of these warehouses are still in existence. Sheds were also built on the quays. Beneath the warehouses a wonderful range of vaults was constructed with fine groined brickwork roofs carried on stone columns. These vaults cover an area of nearly 22 acres and are still in use for the storage of wines and spirits. They can contain nearly 22,000 tons of these commodities in 74,000 casks, to handle which about 28 miles of ground runway rails have been installed. The small Hermitage Basin and lock, the Eastern Dock (650 feet by 420 feet), and the Tobacco Dock between the two larger docks were added in 1815, the general construction being similar. In 1832 the old Shadwell Basin and locks to the Eastern Dock and river were opened. These were constructed by Mr H. R. Palmer, F.R.S., Founder and Vice-President of the Institution. Certain difficulties were experienced with the river wing walls, in connexion with which Rennie and others were consulted. The locks were similar to the Wapping lock and fitted with cast-iron gates which, however, were a failure.

In 1838 a jetty was built in the dock, 780 feet long by 65 feet wide, on cast-iron piles.

The new Shadwell Basin, of 5 acres extent, and locks to the Eastern Dock and river were constructed between 1854 and 1858, Mr James Meadows Rendel, F.R.S., Past-President I.C.E., being the Engineer. The design of the works represents a further stage in the evolution of dock structures, several important modifications having been incorporated.

The gravel was too high, so the foundations were on clay. The north wall, where the clay sloped southwards, was constructed in arcade fashion, with concrete transverse piers built in trenches (see Figs 2, Plate 2). The ground between was excavated to a slope and a brickwork curtain wall was provided at the back of the slope, with brickwork arches and stepped quay wall above between the piers. The other walls were built for half the height in lime concrete with a vertical back and curved front to form a toe ; the remainder of the wall was in brickwork with a stepped back and vertical front. The depth of the dock is 28 feet below T.H.W. The lock entrance is 350 feet long by 60 feet wide with a depth of 28 feet to the centre of the curved invert (see Figs 3, Plate 2). The walls have vertical faces and partly stepped backs ; they are of brickwork on a concrete foundation, the invert being a brick arch on a concrete base with straight granite sills. There are culverts and paddle sluices in the side walls and three pairs of timber gates. The Basin lock is similar except that the inner sill is 7 feet higher.

East India Docks

The East India Dock Company, between 1803 and 1806, built a new dock, 1,410 feet by 560 feet, called the Import Dock, to the north of the old Brunswick Dock, and also an irregular shaped basin with locks to the river and the Import Dock and a passage to the Brunswick Dock, which was altered and re-named the Export Dock (760 feet by 463 feet). The locks were of sufficient size to accommodate the largest East India ships of 1,500 tons burthen. The walls of the new dock were similar to those of the West India Dock.

In 1875, a new river entrance to the basin was constructed, 100 feet long by 64½ feet wide and 31 feet deep to the two sills below T.H.W., and the basin was enlarged.

In 1879, an unfortunate mishap occurred to the south wall of the Import Dock which had been founded on the London clay below the ballast. The wall slipped forward and subsided to some extent. It was subsequently repaired and stabilized with piling and mass-concrete work in front.

During the Second World War, the Import Dock was dried out for the purpose of the construction of "Mulberry Harbour" "Phoenix" units and a similar slip occurred to the west and a part of the south walls. These events emphasize the value and importance of the ballast stratum as a foundation.

St Katharine Docks

Another company, between 1825 and 1828, built the St Katharine Docks immediately to the east of the Tower of London, on land belonging

to the Hospital of St Katharine, a religious establishment founded in 1148. A large area of very poor dwellings and the church of St Katharine, built in 1443, had to be cleared and there was considerable opposition to the scheme.

Thomas Telford, F.R.S., the first President of the Institution, was the Engineer and the work was carried out very expeditiously, although Telford deprecated the haste with which the work was done.

The docks consist of three basins of irregular shape and a lock entrance 180 feet long by 45 feet wide with a central depth over the outer sill of 28 feet below T.H.W. and 24 feet over the inner sill (see Figs 3, Plate 2). The depth of water in the docks was 25 feet below T.H.W. The brickwork walls were similar to those at London Docks except that the back was vertical (see Figs 2, Plate 2), the thickness at the bottom being 10 feet. They were built on gravel with sheet-piling at the toes. The bottom of the dock was covered with puddle clay and Telford evidently feared that the pervious gravel would lead to a serious loss of water from the docks, for at the lock he excavated the gravel and replaced it with puddle clay. This was unfortunate for, although the lock structure has generally remained in a sound state, the gate platforms have become seriously disturbed. The lock structure was of brickwork with curved walls and invert arch, but the gate platforms were in masonry with greenheart sills fastened thereto. Large warehouses were built round the docks extending right up to the quay edge with no open quay space, the front walls being carried on massive cast-iron columns. The idea was to take cargo straight into the warehouses without sorting on the quay and has not been repeated elsewhere. The dock system is now used solely for barge traffic and warehousing. Vaults were built beneath the warehouses.

Surrey Commercial Docks

On the south side of the river, between Rotherhithe and Deptford, there lies a curious maze of irregular-shaped docks, which have developed by stages and a series of reconstructions, carried out by three companies and finally by the Authority. The trade carried on there is principally the importation of softwood with some general cargo and Canadian produce.

The Howland Great Wet Dock has already been mentioned and this was acquired by the Commercial Dock Company in 1807. In 1808 it was reconstructed to be 1,000 feet long, 450 feet wide, and 18½ feet deep below H.W.O.S.T., with timber and lime-concrete walls and a new lock 209 feet long, 42 feet wide, and 18½ feet deep with an elliptical shape (see Figs 3, Plate 2). It was then known as the Greenland Dock. The Norway Dock was also added on the north side. Various ponds for the storage of timber were added to the north and were known as Lady Dock

and Acorn and Lavender ponds; the Lavender Lock provided a small entrance from the river. A number of warehouses were also built.

Another company, the East Country Company built a small dock to the south of the Greenland Dock, with an entrance. This was acquired by the Commercial Dock Company in 1851 and re-built in 1855 as the South Dock; at the same time the Lavender entrance lock was re-built.

The South Dock is 810 feet by 313 feet and the lock 220 feet by 48 feet and 27 feet deep below T.H.W. There was difficulty with the river coffer-dam because the gravel, into which the timber sheet-piles were driven, was very compact and cemented. This condition has sometimes been encountered elsewhere and the formation is known as Blackwall rock. The structures are very substantial, the walls being of brickwork cellular form with concrete filling and backing and masonry facing. The faces of the walls are curved slightly and the backs of the walls vertical without stepping (see Figs 2, Plate 2). The lock invert is a flat arch and the straight sills are of masonry. Wall culverts and sluices were provided. The gates are of timber. A smaller lock communicated with the Greenland Dock.

In 1801, a third company, the Grand Surrey Canal Company, who later added " and Docks " to the title, built the Grand Surrey Canal from a point near the present Surrey Entrance Lock southwards through what are now known as the Stave Dock and Russia Dock towards Deptford, whence it turns westwards to Camberwell with a branch to the south to Peckham. This project was intended to continue to meet the River at Vauxhall and thence to Croydon and even to Portsmouth!

Various timber ponds known as the Canada, Albion, Quebec, and Centre Ponds, and the Commercial Basin, were added to the west of that part of the Canal which was enlarged into the Stave and Russia Docks.

In 1860, the Main or Albion Dock (1,200 feet by 450 feet and 27 feet deep) and the Surrey Lock and Basin were added, linking with the Stave Dock. None of these ponds calls for any special mention, since they were comparatively small with timber campshedding at the sides. The basin and dock walls were battered on the face.

The Surrey Lock, built in 1860 by Mr G. P. Bidder, Past-President I.C.E., and Mr Jennings, is 250 feet long, 50 feet wide, and 27 feet deep below T.H.W. (see Figs 3, Plate 2). The design was robust and simple, the vertical walls and invert arch of the lock being in brickwork with concrete backing. The lock gates were of wrought iron.

In 1863, the two companies amalgamated as the Surrey Commercial Docks Co., and in 1876 the Canada Dock was added, with a passage to the Albion Dock. In connexion with this work parts of the Canada and Albion Ponds were absorbed and filled-in. The Canada Dock is 1,500 feet long, 500 feet wide, and 27 feet deep, and the Albion passage is 50 feet wide. The walls on the west, south, and north sides were of fairly modern and substantial design in brickwork and concrete (see Figs 2,

Plate 2), with a portion at the north end of arcade form; they were founded on the gravel, but at the south end the gravel was penetrated and the walls were founded much lower on the Thanet sand, which gave trouble through running. The face of the wall was curved in section and the back was stepped.

In 1894, the first stage of the entire reconstruction of the Greenland Dock was undertaken. A passage, 60 feet wide and 27 feet deep below T.H.W., fitted with gates, was constructed at the south-east end of the Canada Dock and a new dock was built eastwards towards the old Greenland Dock. In 1898 the latter was entirely reconstructed and joined with the first stage, the complete dock then being 2,350 feet long, 450 to 370 feet wide, and 31½ feet deep below T.H.W. (see Figs 2, Plate 2). The lock gates were operated by hydraulic rams—the first instance of the use of such machinery in the port.

When building the new river entrance-lock, 550 feet long, 80 feet wide, and 35¼ feet deep to the centre of the three sills, considerable difficulty was experienced with the foundations, owing to running sands and blows from the river beneath the dam. Part of the work was carried out by using monoliths to form the outer base. Sir John Wolfe Barry, K.C.B., LL.D., F.R.S., Past-President I.C.E., was the Engineer for the latter portion of the works, which were completed in 1904, Messrs S. Pearson and Sons Ltd. being the Contractors.

The passage to the Norway Dock and the lock to the South Dock were retained, the passage to the Russia Dock was enlarged, and a new barge lock was constructed to connect with the Surrey Canal, the water level of which is 3 feet below that of the dock at T.H.W. These passages, two on each side of the dock, unfortunately divide up the north and south quays and therefore impose a limitation upon the lengths of the berths. Until recent years, each length of quay was sufficient for either one or two vessels, but with the growth in the length of ships the tendency is for this to apply no longer and to some extent, therefore, the full economic use of them is now difficult to secure.

Millwall Dock

In 1864, the Millwall Dock Company was formed and obtained an Act to construct a dock in the Isle of Dogs, south of the West India Docks, on a vacant tract of low-lying marsh land.

Mr John Fowler and Mr William Wilson were the Engineers and Messrs Kelk and Aird the Contractors, the works being completed in 1868. It was intended that the dock should be of inverted T shape formed by an east-to-west dock across the peninsular, with an entrance at each end and a branch dock in the middle running northwards. In fact, the eastern portion and entrance were not constructed and, in order to provide for vehicular traffic crossing the dock, two arms were built with a passage

and draw-bridge, so forming an Inner and an Outer Dock. This has proved to be an awkward obstruction. The lock entrance, nearly opposite the Greenland Entrance, was 450 feet long by 80 feet wide and 28 feet deep below T.H.W. to a curved arch-invert sill. A walled bellmouth was provided at the river end. Three pairs of double-skin wrought-iron gates, operated by chains and hydraulic engines, were provided. The double skins were intended to provide greater strength but not buoyancy. The lock walls have vertical faces and are of brickwork and lime-concrete construction, the invert being in brickwork, with greenheart sills fastened to masonry. The culverts and sluices are in the walls. The dock walls have a battered face and stepped backs and these also are in brickwork and lime concrete, the brickwork forming a facing wall and horizontal layers in the body of the wall (see Figs 2, Plate 2). At this stage of dock construction, brickwork and masonry were still regarded as the principal structural materials and concrete, which was not yet made with Portland cement, was looked upon as merely stiff filling. The dock is 350 feet wide, the east-west portion being 2,000 feet and the north-south portion 2,250 feet long. An important feature was the incorporation, for the first time in the Thames impounded docks, of a dry dock. This was placed on the south side of Outer Dock and was made 413 feet long, with an entrance width of 65 feet and a cope width of 90 feet, the depth over blocks being 22 feet 11 inches below T.H.W. A wrought-iron ship caisson was provided to fit up to stops. A bucket-pump, worked by hydraulic power in a well and lifting 14 tons of water at a stroke, was used for emptying the dry dock.

The foundations of the entrance, walls, and dry dock were partly on the ballast and, where this was too high, on the Woolwich and Reading bed. Railways were provided, since they had by this time been laid down in the adjoining West India Docks, but the sheds and warehouses were small and low roofed.

Victoria Dock

In 1838 the steamship Great Western (2,300 tons) made the voyage to America; and by about this time the congestion of shipping in the river was again increasing, and the smaller entrances of the older docks were becoming of less use. By the middle of the century railways had become established and the value of railway connexions at docks was becoming obvious. And so, in 1854, the Victoria Dock Company was formed to carry out a bold scheme of constructing a large dock farther down the river on vacant marshes at West Ham. The land was cheap and the ingenious design of some of the works made the enterprise a very economical one. Although the works proved to have but a limited life they present a classical case of this form of economy applied to dock construction. The total cost was £940,000.

The engineer for the new dock was Mr G. P. Bidder. The dock and tidal basin were made 4,050 feet long, 1,050 feet wide, and 24 feet deep below T.H.W. The entrance lock at the western end was made 325 feet long between the two pairs of wrought-iron gates, 80 feet wide, and 28 feet deep below T.H.W. at the outer sill and 25 feet 6 inches at the inner sill, the sills being made practically level (see Figs 3, Plate 2). A bellmouth entrance was provided from the river and a short passage spanned by a road-and-railway swingbridge connected with the tidal basin.

A novel feature of the design of the dock was the construction of four solid jetties on the north side, 581 feet long and 140 feet wide, spaced 430 feet apart, as well as the two solid jetties separating the dock and the basin, which had a passage between them 80 feet wide and 25 feet 6 inches deep below T.H.W. to a level sill fitted with gates.

The total length of berthing accommodation was about 3 miles and this dock became very popular and useful for shipowners.

At the lock, the gate platforms and recessed walls, which were built in brickwork, were founded on the London clay beneath the ballast. The other parts of the lock walls were constructed of cast-iron T-shaped piles driven on a batter a few feet into the clay, with cast-iron plates between them. A lime-concrete wall was built behind the plates and piles, and the latter were tied back with wrought-iron ties. The upper portion of the walls was in brickwork.

Unfortunately, as at the St Katharine Dock entrance, the whole of the gravel was taken out and puddle clay substituted in the floor of the lock. A layer of puddle clay, 2 feet thick, was similarly placed over the entire bottom of the dock and basin on the ballast. The fear of losing water from the impounded docks to the river through the pervious ballast seems to have influenced certain of the early engineers, but the removal of the ballast, which is a first-class foundation material, and the substitution of puddle clay, which is not, sometimes proved to be a remedy far worse than the disease. In this case, the ground-water had been kept at a low level by pumping from sumps during the construction of the works, but when pumping ceased and the water rose, the puddle floor of the lock chamber lifted and softened and the walls subsided and moved forward to some extent. The work was expeditiously repaired, the piles being driven down farther, the walls rebuilt, and the puddle clay replaced by a lime-concrete invert.

The jetties were also built with cast-iron T-section piles with horizontal brickwork arches between them, backed with lime concrete founded on the ballast (see Figs 2, Plate 2). Brickwork vaults were constructed in the jetties and the T-section piles were tied back to the vault walls with wrought-iron tie-rods. Double-storey brickwork sheds were built on the jetties over the vaults and several large multi-storey warehouses were built from time to time along the north side of the dock, which was served by both railways and a dock road.

TABLE 1.—PARTICULARS OF DOCKS—PORT OF LONDON AUTHORITY

Dock group	Distance below London Bridge: miles	Totals			Docks	Max. depth: feet	Max. W.L. above T.H.W: feet	Lock entrance			Dry docks		
		Area of estate: acres	Water area: acres	Length of quays: miles				Length: feet	Min. width: feet	Depth to centre of sill below T.H.W: feet	Length: feet	Min. width: feet	Max. depth over blocks: feet
St Katharine .	0·66	25	10·10	0·94	Basin Western Eastern	18 18 18		180	45	{28 outer 24 inner			
London . .	0·86 to 1·94	100·25	34·67	2·97	Hermitage Basin Wapping Basin Western Tobacco Eastern Shadwell Basin	25·25 23 23·75 24·75 24·75 25 and 28	3·75 3·75 3·75 3·75 3·75 3·75	170 350	40 60	23 28			
Surrey Commercial	1·81 to 3·56	381·50	134·48	8·88	Basin Albion Canada Quebec Russia Island Stave Lavender Lady Norway Greenland South	27 27 27 27 19 23 19 and 21 19 19 19 31·5 27		250 550 220	50 80 48	27·25 35·25 27	88·25	22·5	7·00

West India and Millwall	2·80 to 6·31	466	132·88	6·71	Import	26 and 29							
					Blackwall Basin	26		480	60	30			
					Export	26							
					Junction	25							
					South Millwall	29		584	80	35			
					Inner	29							
					Outer	28		450	80	28	546·42	65	22·12
East India .	6·71	49	23·34	1·16	Basin	32		100	64·5	31			
					Import	24	2						
Royals . .	7·10 to 10·59	1112·5	235·85	11·05	R. Victoria	28 and 31	2·5	325	80	20·5			
					R. Albert	29·5 and 34	2·5				574·5	80	22·75
											500	62	22·75
					do. Basin	34·5	2·5	550	80	30			
								550	80	36			
					K.G.V.	38	2·5	800	100	45	750	100	35
Tilbury . .	24·78 to 26·22	725	103·79	3·95	Basin	Tidal					260·5	59·83	26·83 (T.H.W.)
											119	70	31·75 (T.H.W.)
											560	59·83	26·83
					Main	38		695	79·5	{44 outer 38 inner	698·3	70	31·08
											752·3	110	37·5
					East Branch	38							
					Centre Branch	38							
					West Branch	38							
					Main Dk. Extension	42·5		1,000	110	45·5			
Totals		2,859·25	675·11	35·66									

Hydraulic power was installed for the first time to operate the gate, sluice, and bridge machinery and also the quay cranes. This was copied by the other dock companies in due course and became the universal method of operating such machinery for many years. The hydraulic power was generated by steam engines and low-speed direct-acting pumps.

Amalgamation of Companies

It is now necessary to digress for a moment to mention a new aspect of the commercial development of the port.

In 1838, the West India Dock Company, which had acquired the City Canal in 1829, combined with the East India Dock Company into the East and West India Dock Company, and I have already referred to the amalgamation, in 1863, of the Surrey Companies. Increasing competition between the various companies and also with the growing number of riverside wharfs forced other amalgamations and, in 1864, the London, St Katharine, and Victoria Companies combined into the London and St Katharine Dock Company.

A change was also taking place in the nature of commerce, owing to the growth of free trade. The number of classes of dutiable goods, which, in 1842, was 1,052, had been reduced to only 48 in 1860 and this led to a reduced demand for bonded warehouse space and an increased demand for quay-shed or transit space.

Royal Albert Dock

The immediate success of the Royal Victoria Dock, as it became termed, and the general growth of the size of shipping and of the volume of trade led the London and St Katharine Dock Company to undertake a very large extension eastwards to the river at Gallions Reach, known as the Royal Albert Dock.

This large work constructed between 1874 and 1880 at a cost of £2,200,000, and probably the most valuable and useful in the history of the port until recent times, consisted of a dock with straight quays on each side as distinct from the jetty system of its neighbour. Although opinions differ somewhat upon the merits of the two systems of dock lay-out, the long straight quay has proved its value and flexibility in London and has been adopted in most later works; the jetty system, which is not so flexible and which is more costly, has been superseded or has become to a large extent obsolescent.

The dock, which lay between Connaught Road and Manor Way, was made 6,600 feet long and 490 feet wide, with a depth of 27 feet below T.H.W. At the western end a passage, 80 feet wide and 25½ feet deep, connected with the Royal Victoria Dock. The North Woolwich Branch of the Great Eastern Railway crossed the site of this passage and had

to be diverted into tunnels built within the invert together with subways for mains; a road-and-railway swing-bridge spanned the passage. At the eastern end, the dock was connected to a tidal basin by means of a passage, 80 feet wide and 27 feet 3 inches deep, provided with a pair of gates and also with a road swing-bridge. The lock entrance was made 550 feet long, 80 feet wide, and 30 feet deep below T.H.W. to three partly level sills, and three pairs of single-skin wrought-iron gates were installed. The lock had a bellmouth entrance from the river with heavy timber leading-in jetties, which at the time were a new feature. Water was kept impounded in the dock by means of a steam pumping-station adjacent to the lock.

The dock walls are of modern design (Figs 2, Plate 2) with a sloping base, projecting toe, and stepped back, and were built of Portland-cement concrete—the first instance of the use of this material for such works in the port. Two dry docks were built on the south side at the western end, aligned at an angle with the dock, the dimensions being:—

Western Dry Dock: 408½ feet long, 56½ feet wide at entrance, and 80 feet wide at cope.

Eastern Dry Dock: 500 feet long, 62 feet wide at entrance, and 85 feet wide at cope.

Depth over blocks of both docks: 20¼ feet below T.H.W.

Both docks were provided with ship-type caissons.

Single-storey transit sheds, 120 feet wide and 300 to 336 feet long, were built along both sides of the dock. They had cast-iron columns and wrought-iron roof trusses and were sheeted with corrugated iron. They were illuminated by the first electric lighting installation in the port.

The railway arrangements at the back of the sheds—four lines on the north side and three on the south—were excellent, and vehicular roads were also provided. The quays were 40 feet wide and were also provided with railways and quay-crane tracks.

The dock was, at the time, the longest in the world and the following particulars give an idea of the magnitude of the work. Four million cubic yards of material was excavated by steam navvies and land dredgers. 43 million gallons of water was pumped from the works per day. 500,000 cubic yards of concrete was used (including 80,000 tons of Portland cement) and 20 million bricks. Between 2,000 and 3,000 men were employed. At the lock entrance, the walls were faced with brickwork, which, however, has not proved a successful feature, and the sills and quoins were of granite. The gates were chain-operated by hydraulic power.

It was soon found that the depth at the lock was inadequate for the increasing size of ships and, in 1887, a second lock entrance was built just north of the first and of similar size and design, except that the depth

was made 36 feet below T.H.W. (Figs 3, Plate 2). The basin was correspondingly enlarged.

The largest ship trading to London in 1880 was the 4,457-ton " Queen " of the National Line ; the new entrance is capable of accommodating ships up to 12,000 tons and could, in fact, take the largest ships trading to the port up to 1903, an instance of remarkable foresight.

The engineer for the works was Sir Alexander Meadows Rendel, the Contractors being Messrs Lucas and Aird.

Tilbury Docks

The success of the Royal Victoria and Royal Albert Docks of the London and St Katharine Docks Company was not matched by the East and West India Docks Company, whose trade continued to decline. Notwithstanding that their docks were nearer to the City and its markets, and that the berthing accommodation was first class, the extensive warehouses were not suited to the rapid transit of goods and the railway facilities were inferior to those of the Royal Albert Dock. The chief defect, however, was the small and quite inadequate entrance locks and it was clear that the Company had undoubtedly made a great mistake in not building a much larger entrance to the South Dock. If that had been done, or if, between 1870 and 1890, it had been rebuilt together with better transit facilities, the South Dock might have captured some of the trade that went to the rival docks lower down the river.

The Company eventually decided, in 1882, to try and recover their position and embarked upon a bold scheme for building still larger docks farther down the river. They chose Tilbury as the site, for it was considered that facilities there would be better placed for very large vessels as well as more convenient and attractive for the growing passenger traffic to and from the East (then being handled in the Royal Albert Basin).

Mr A. Manning and Mr D. S. Bayne were appointed Engineers and Messrs Kirk and Randall were awarded the contract.

The design comprised three branch docks, 1,600 feet long and 300 feet wide, the eastern and western branches tapering to only 200 feet wide at the head, the peninsulars between the branch docks being 400 feet wide. A main dock, 1,780 feet long and 600 feet wide, connected the branch docks at right angles to them. At right angles to the main dock, a lock entrance was built, 700 feet long and 80 feet wide, with a depth to the outer and middle sills of 44 feet, and to the inner sill of 38 feet below T.H.W. (Figs 3, Plate 2). The vertical walls of the lock were faced with brickwork and the invert was a brick arch on concrete. Wrought-iron tank gates were provided. Between the lock and the river, an open tidal basin was excavated to a depth of 45 feet below T.H.W., with bellmouth leading-in timber jetties to enable large ships to enter

still water out of the tidal stream before making the entrance lock. Two tidal berths were built in the basin to enable passengers to embark or disembark without delay. Adjacent and parallel to the lock, two dry docks were constructed, each 846 feet long, one 70 feet wide and 32 feet deep below T.H.W., and the other 60 feet wide and 27 feet deep below T.H.W. They were open at each end, so that vessels could enter either from the basin or from the dock, and were provided with ship caissons at both ends as well as middle caissons which could be placed in intermediate grooves, so making four dry docks of variable length. However, the system proved less practicable than ingenious, and permanent intermediate dams were substituted after a time, so making four docks of fixed length. Steam-driven centrifugal pumps were installed.

The ground over the site of the docks was excavated in the open for a depth of about 10 feet below its original level of about 7 feet below T.H.W. The river bank was temporarily retained across the mouth of the basin as a dam, and the area behind was kept dry by pumping from sumps sunk to the ballast. The effect of this pumping was to lower the head of water in the ballast for a considerable distance, as had also been experienced in other dock works, and in consequence the very soft wet marsh clay and peat was drained to some extent.

The dock walls, which had a slightly battered face and stepped backs, a projecting toe, and a sloping base founded on the ballast (Figs 2, Plate 2), were constructed in timbered trenches sunk below the preliminary site excavation and the depth in the docks was made 38 feet below T.H.W. In one or two places, where the ballast was at a lower level, the walls were founded on timber piles. The quays were provided with single-storey transit sheds, 300 feet by 120 feet, with railways on the quays and at the back of the sheds. There was, however, very little provision for vehicular traffic except for access to the quays. The latter were equipped with hydraulic cranes and electric lighting was installed in the sheds.

The docks, which were opened in 1886, were, for a time, unsuccessful. The original estimate of £1,100,000 was greatly exceeded, mainly because of engineering difficulties in connexion with the excavation of the soft material. The original contract was terminated before the completion of the work, which was eventually carried out by Messrs Lucas and Aird, the total cost being £2,800,000.

Although the depth in the docks was in advance of anything that had yet been provided in the port (indeed, it proved for many years to be excessive), the branch docks were made much too narrow.

There is little doubt that the open tidal basin, whilst affording an area of quiet water for ships approaching the lock, was a bad mistake and was of the wrong shape, for it has, in fact, proved to be a perpetual mud trap involving very great expense in dredging ever since. The solution

of this problem is now being carefully studied. From the traffic aspect, shipping continued to prefer the docks farther up the river and Tilbury proved to be too far from London for the economical handling of cargo, so much of which in London is transferred to lighters for transport to wharfs. It seems probable that the provision at Tilbury of a riverside landing stage or quay for the use of passengers, together with the rebuilding of the entrance to the South West India Dock with much larger dimensions would have been of much greater benefit to the Company.

Amalgamation

In 1888, the two principal Companies came to an arrangement and instituted a Joint Committee of management; in 1900, there followed amalgamation as the London and India Docks Company. In 1894, in an effort to improve access to the West India Docks, the Committee reconstructed the old river ship entrance at Blackwall, the new lock being 480 feet long and 60 feet wide with a depth of 30 feet below T.H.W. The passages to the Import and Export Docks were similarly reconstructed. In 1900, the South West India Lock was also lengthened to 480 feet and deepened to 29 feet. These works, whilst enabling vessels of 8,000 tons gross to enter, were obviously too restricted in view of the larger entrances at the Royal Albert Dock.

Cold Storage

Some time before this a new import had begun to develop. The colonization of New Zealand, particularly, and the rearing of large numbers of sheep on the Canterbury Plains led to the development of ships equipped with refrigerated holds, in which lamb and mutton could be successfully transported to the home market. This, in turn, led to the provision of cold stores at the docks and several of these were built at the Royal Victoria Dock and at Smithfield Market, as well as adaptations at West India Dock.

Twentieth Century Extensions

I have now traced briefly the evolution of the dock systems and some of the salient features connected with their construction up to the end of the nineteenth century; at about this time, however, it became apparent that there would be great financial difficulty in securing further dock expansion by the Companies. Further, in order to render any future development of value, and indeed to obtain the best use of existing accommodation as well as to cater for larger shipping, it would be necessary to deepen the river channels.

Shipping was growing rapidly in size and an era of dock expansion on

an even larger scale than before was about to begin in all the major ports of the United Kingdom as well as abroad. In 1900, this matter came before Parliament. The Government attitude was represented by the President of the Board of Trade, Mr C. T. Ritchie, afterwards Lord Ritchie of Dundee, the result being the appointment of a Royal Commission of Inquiry. The eventual outcome was the constitution by Act of Parliament of the Port of London Authority, which, in 1909, under the Chairmanship of the Rt Hon. Viscount Devonport, P.C., took over the assets of the Dock Companies and assumed the management of the port, including the docks and the river below Teddington, the latter having been formerly vested in the Thames Conservancy.

The first Chief Engineer of the Port, Mr (later Sir) Frederick Palmer, K.C.M.G., C.I.E., Past-President I.C.E., immediately formulated outline schemes for dock improvement and new construction, which, in large measure, have been followed broadly during the ensuing years, and which were complementary to the extensive deepening of the river and estuary down to the Nore, by dredging.

Various improvements were soon put in hand. At London Docks, the passage between the Western and Eastern Docks was widened, the jetty in the Western Dock was reconstructed in reinforced concrete with a double-storey warehouse, and new double-storey brick and reinforced-concrete warehouses were built on the north quay, which was widened. The Hermitage Entrance was closed and an impounding pumping station built therein, enabling the water level in the dock to be raised to 3 feet 9 inches above T.H.W.

At the West India Import and Export Docks the north quays were widened (see Figs 2, Plate 2) by 56 feet and 20 feet respectively, by the construction of open or " false " quays consisting of reinforced-concrete cylinders with piles driven therein surmounted by reinforced-concrete beam-and-slab decking, with precast bracing set into the old walls. This enabled the docks to be deepened by 3 feet to 26 feet below T.H.W., the bottom sloping down under the open work. Since the old walls were founded on the ballast their stability was not endangered by this procedure, which has since been repeated elsewhere and which has proved a remarkably effective and economical method of achieving greater depth in the docks without reconstructing the older walls. The ballast once again proved its immense value as a foundation material.

New brick and reinforced-concrete warehouses were built on the north quay of the Import Dock, partly over the false quay, and new steel transit sheds were erected.

At the East India Import Dock false quays and new sheds were also built and the lock from the basin was reconstructed, 300 feet long, 80 feet wide, and 31 feet deep below T.H.W. A new impounding pumping station raised the water level in the Import Dock by 2 feet.

The Millwall dry dock was lengthened to 546 feet.

At the Royal Albert Dock, a new impounding pumping station was built and the water level in this dock and the Royal Victoria Dock raised by 2 feet 6 inches. The Western Dry Dock was lengthened to 574 feet 6 inches and widened to the minimum of 82 feet at the entrance. On the north side of the Royal Albert Dock the first electric quay cranes were installed.

At Tilbury Docks, where the demand for berthing accommodation had increased, the first stage of a scheme of extension westward of the Main Dock was undertaken. The south quay was lengthened by 1,531 feet with a return section at the west end. The dock was correspondingly lengthened, with a depth of 42½ feet and a bottom width of 300 feet. The very soft marsh clay, which could not be drained owing to the proximity of the existing dock, made it impossible to sink trenches from the surface, within which to construct the ordinary type of gravity dock wall. This was therefore formed with monoliths 30 feet square, each with four wells, built from mass-concrete blocks and sunk to the ballast; the back wells were filled with concrete and the front ones left empty (see Figs 2, Plate 2). The new quay, which is 40 feet wide, was equipped with three steel and corrugated-iron transit sheds, 120 feet wide and about 600 feet long, with a continuous railway platform at the back, and also with electric quay cranes. The Contractors for the works were Topham, Jones and Railton Ltd.

The largest of the dock extensions undertaken by the Port Authority was the construction of the King George V Dock, to the south of the Royal Albert Dock, and connected thereto by a cutting. This magnificent dock was commenced in 1912, the Contractors being S. Pearson and Sons. In the same year, Mr (later Sir) Cyril Kirkpatrick, Past-President I.C.E., succeeded Mr Palmer as Chief Engineer, and Mr Palmer was retained as consultant to the Port Authority.

The dock is 4,500 feet long, the width being 630 feet at the eastern end tapering to 450 feet at the western end, and the depth is 38 feet below impounded level, which is 2½ feet above T.H.W. At the eastern end, a new lock entrance was built, 800 feet long, 100 feet wide, and 45 feet deep below T.H.W. to the three sills (Figs 3, Plate 2), with a large bellmouth approach from the river formed beyond the lock walls by two massive timber leading-in jetties. The lock is provided with buoyant steel gates fitted with crocodiles and rollers and worked by hydraulic rams.

The lock structure and the dock walls were of mass concrete and founded on the ballast or the chalk. At the western end, a dry dock was built, 750 feet long, 100 feet wide, and 35 feet deep below impounded level. The dry dock was equipped with a floating ship-type caisson, an electrically driven pumping station, and a 25-ton dock-side crane.

On the south side of the dock seven reinforced-concrete dolphins, 500 feet long, were constructed to carry electric quay cranes; they left

a water space 32 feet wide between the wall and the back of the jetties to accommodate barges (Figs 2, Plate 2). On the north side, six double-storey warehouses were built of brick and reinforced concrete, with steel roofs and under-slung electric travelling cranes. On the quay are electric cargo cranes. Seven single-storey steel transit sheds were built on the south quay, and the entire dock is served by railways and vehicular roads. The First World War occasioned much delay and the dock was finally completed by the Port Authority in 1922, the total cost being £4,130,000.

At Tilbury, a cargo jetty was constructed in the river, 1,000 feet long and 50 feet wide, on three rows of reinforced-concrete cylinders with piles therein, connected by precast bracing and two heavy reinforced-concrete decks, the space between being used as a transit shed. Electric quay cranes are mounted on the top deck, which also carries two railways. The jetty, which was completed in 1921, is connected with the shore by a curved approach. At the outer side a depth of 50½ feet below T.H.W. has been provided and the inner side is used by small vessels and barges.

At the western end of the north side of the Royal Albert Dock, three of the transit sheds were replaced in 1920 by a double-storey brick and reinforced-concrete building with a steel roof, 1,100 feet long. The ground floor is used for transit purposes and the upper floor, which is divided into four sections, is insulated and refrigerated. Adjacent to this, a large six-storey brick and reinforced-concrete cold store was built in 1918. The total refrigerated space at the Royal Docks was then increased to about 4 million cubic feet, which is equivalent to a storage capacity of nearly 1 million carcases of mutton.

The dock-side cold floor was intended for sorting prior to conveyance to the store on elevated bands, but it was found to be more economical to sort to rail wagon for conveyance to the store. The upper floor of the shed was not therefore used as refrigerated space to any extent until the Second World War and since.

Very large warehouses were also built on the north side of the Royal Victoria Dock, between 1920 and 1924, for the storage of tobacco, and two berths were reconstructed on cylinders and piles and equipped to handle chilled beef.

At the Surrey Commercial Docks, a scheme was formulated for a large new dock to cross the middle of the estate and, possibly, a new entrance lock some distance above the Greenland Entrance. The first stage of this, the Quebec Dock, was undertaken in 1923 on the site of the Quebec, Canada, and Centre ponds.

An 80-foot-wide cutting was formed from the east side of the Canada Dock and a triangular shaped dock was constructed between this point and the Russia Dock. A north wall, 1,300 feet long, and a south wall, 900 feet long, were built, the eastern end being left as a slope to allow for future extension. The walls were all founded deep enough to permit the

cutting and dock to be deepened to 35 feet, but the depth was restricted to 27 feet in the first instance. Large timber-storage sheds were built similar to those previously built at the Canada Dock and elsewhere. These have open ends and brickwork fire division walls with light steel roofs, and enable timber to be stacked to a height of about 25 feet.

The Lavender and Acorn Ponds were reconstructed and deepened to 19 feet below T.H.W., with a steel sheet-piled wall on the east side, 1,670 feet long. The Stave Dock—Lavender Dock passage was widened to 55 feet, and deepened to 23 feet. The old Lavender Entrance was closed and an impounding station built therein.

In 1925, Lord Devonport retired and was succeeded as Chairman of the Port Authority by the Rt Hon. Lord Ritchie of Dundee, whose father had been so prominently connected with its inception.

In 1926, a large scheme of development at Tilbury Docks was put in hand, consisting of the construction of a new entrance lock at the western end of the Main Dock, which was also widened, and of a new dry dock, at a total cost of £2,460,000. The lock was made 1,000 feet long, 110 feet wide, and 45 feet 6 inches deep to the three sills below T.H.W. (see Figs 3, Plate 2), and was equipped with three pairs of ram-operated tank gates. The lock was aligned at an angle with the river—unlike all the other entrances of the Authority's docks (excepting Shadwell and Surrey), which are almost at right angles to the line of the river. Massive timber leading-in jetties were also built to form a bellmouth entrance.

The dry dock at the eastern end of the Main Dock was also set at an angle to facilitate the entry of large ships and was made 750 feet long, 110 feet wide at the entrance, and 37½ feet deep below T.H.W.

These structures were formed by means of monoliths, which have proved to be a successful mode of construction under the conditions prevailing at Tilbury. New impounding pumps were installed in conjunction with the dry-dock pumps.

The Port Authority also provided excellent new facilities for passengers by means of a floating landing stage, 1,142 feet long, moored in the river to dolphins immediately below the entrance to Tilbury Basin, together with a large baggage hall for Customs examination, adjacent to Tilbury Riverside Railway Station. A length of 300 feet of the stage is segregated for cross-river ferries. Covered gangway bridges connect the stage with the hall. Passenger liners, at whichever of the dock systems they may berth, now call regularly at the stage on passage both inwards and outwards.

At the same time, at the West India Docks, the South Dock entrance at the eastern end was entirely reconstructed, the new lock being made 584 feet long, 80 feet wide, and 35 feet deep to the three sills below T.H.W. New passages were also constructed to connect the Import, Export, and South Docks, and the latter with the Millwall Inner Dock; a new impounding pumping station was installed at the west end of the

Table 2.—tidal data

Place	Relative to London Bridge		H.W.O.S.T.		H.W.O.N.T.		L.W.O.N.T.		L.W.O.S.T.		Highest recorded tide		Lowest recorded tide	
	Distance: miles	Time of H.W. hrs mins	Newlyn	T.H.W.	Newlyn	T.H.W.	Newlyn	T.H.W.	Newlyn	T.H.W.	Newlyn	T.H.W.	Newlyn	T.H.W.
Southend Pier	43·16	− 1 17	+ 8·97	− 2·13	+ 5·75	− 5·35	− 5·10	− 16·20	− 7·85	− 18·95	+ 15·12	+ 4·02	− 12·02	− 23·12
Tilbury Lower Entrance	25·91	− 0 51	+ 10·24	− 1·16	+ 6·83	− 4·57	− 5·61	− 17·01	− 8·59	− 19·99	+ 16·08	+ 4·68	− 12·6	− 24·00
King George V Entrance	10·33	− 0 26	+ 11·46	+ 0·06	+ 7·69	− 3·71	− 6·09	− 17·49	− 9·18	− 20·58	+ 16·90	+ 5·50	− 12·6	− 24·00
S.W. India Dock Entrance	6·13	− 0 16	+ 11·72	+ 0·32	+ 7·95	− 3·45	− 6·08	− 17·48	− 9·15	− 20·55	—	—	—	—
Greenland Entrance	3·46	− 0 9	+ 11·80	+ 0·40	+ 8·04	− 3·36	− 6·09	− 17·49	− 9·16	− 20·56	+ 17·40	+ 6·00	—	—
London Bridge	—	—	+ 12·10	+ 0·70	+ 8·34	− 3·06	− 5·93	− 17·33	− 8·90	− 20·30	+ 17·75	+ 6·35	− 13·2	− 24·60
P.L.A. landward limit	18·45	+ 1 0	—	—	—	—	—	—	—	—	—	—	—	—
Teddington Lock	18·62	+ 1 1	+ 14·00	+ 2·60	+ 9·83	− 1·57	†	—	†	—	+ 17·35	+ 5·95	—	*

* No record for Teddington Lock, but at Richmond Lock — 3·5 Newlyn.
† Maintained at L.W. Level of + 5·6 Newlyn.

South Dock. All these works were carried out in sheet-piled and timbered trenches with mass concrete construction and cost, in the aggregate, £1,650,000.

When the late Mr Asa Binns, President-Elect in 1946, was Chief Engineer, further schemes of improvement were formulated and some of the works put in hand. The principal of them were at the Royal Docks, where the Royal Albert Dock was deepened to 34 feet below impounded level, except at the south quay and a false quay, 19 feet wide, was constructed on the north side (see Figs 2, Plate 2).

The Royal Victoria Dock was almost entirely remodelled. Its depth was increased to 31 feet, including the invert of the cutting between the Victoria and Albert Dock; a new quay of open-cylinder and sheet-piled type was constructed at the western end of the south side to replace a dilapidated timber wharf, and at the eastern end the dock was widened and two new monolith quays were built in conjunction with the erection of a large flour mill by tenants. Three other large flour mills, with dolphin or quay-wall berths, had, in pre-war years, also been built by tenants on the south side of this dock.

On the north side, the old solid jetties were beginning to fail by the corrosion of their tie-rods and the sheds had long proved inadequate. In consequence the jetties were cut back and a new straight open-type quay, 3,300 feet long, was built. It consisted of three rows of reinforced-concrete cylinders with piles and a heavy beam-and-slab superstructure with sheet-piling at the back (see Figs 2, Plate 2). The spaces behind this quay were reclaimed by pumping with ballast dredged from the dock, and on the ground thus formed five large three-storey warehouses were built together with railways and a new vehicular road. The total floor area of these warehouses is 26 acres.

The total cost of these improvements was £1,850,000 and various similar but smaller improvements were carried out at the West India, Millwall, and Surrey Commercial Docks.

The war put a stop to further improvements and indeed was the cause of an immense amount of wide-spread damage and destruction. Much of this was the result of fire among the older timber-floored warehouses at the upper docks and the timber in store at the Surrey Commercial Docks; there was also a great deal of damage by high-explosive.

In 1939, Lord Ritchie retired, and was succeeded by the Rt Hon. Thomas Wiles, P.C. The present Chairman of the Port Authority, the Rt Hon. Viscount Waverley, P.C., G.C.B., G.C.S.I., G.C.I.E., F.R.S., an Honorary Member of the Institution, took office in 1946.

Very rapid strides have been made with the repair and reconstruction of the docks and buildings and the opportunity has of course been taken to do so on the most up-to-date lines and to effect as much improvement as possible.

Besides this, further large works of development are in hand, prin-

pally the reconstruction of the lower entrance lock of the Royal Albert ock, the construction of a new quay and shed on the north side of the ain Dock at Tilbury (Figs 2, Plate 2) and of warehouses at West dia Dock, and the widening of the Canada-Greenland cutting at Surrey ommercial Docks.

Analysis

Finally, I would like to make a few remarks regarding the size and owth of the shipping for which all these vast dock works have been onstructed and which will be the all-important factor in the design of ture works.

The growth and magnitude of the shipping and trade of the Port of ondon, and in particular that with which the Authority's Docks have ad to deal, are shown in Table 3.

In 1794, 3,663 ships entered from abroad, the tonnage being 620,000 nd, during the following year, 11,964 coastwise with a tonnage of ,176,000. The largest vessels using the port at the commencement of he nineteenth century were East Indiamen of 1,550 tons burthen, 199 feet ong and 43½ feet beam, but they were exceptional and the majority of he vessels were of 300–400 tons burthen. Ship sizes tended to decrease fter the end of the Napoleonic Wars and it was not until 1838 that a arger ship was built, this being the transatlantic wooden paddle-steamer British Queen "—2,016 tons, 234 feet long and 37½ feet beam. In 1853 he P. & O. Company launched their iron screw-steamer " Himalaya "—,437 tons, 340 feet long, and 44½ feet beam.

During the second half of the nineteenth century, the size of shipping ncreased steadily and so consequently did that of the lock entrances vhich were required. Comparison of the tonnage of old vessels with hips of today is made difficult by the changes in the rules of measure-nent, for until 1835 tonnage was calculated on an empirical formula vhich disregarded both depth and fineness; from 1836 to 1854 a simple olumetric rule was in use and it was not until 1855 that the basis of the resent volumetric system was established. Figs 4, Plate 2, shows the limensions of certain of the largest ships at various dates since then nd also of certain of the largest ships for which the Port of London has ad to provide during recent years.

The diagram also shows the dimensions of the lock entrances con-tructed at the docks during the same period, together with those of the argest locks built for commercial shipping throughout the world.

It will be seen that the London entrances have, except for the very argest north Atlantic passenger liners, always been not only adequate or the world's largest mercantile ships which have traded to London but ave provided a considerable margin at the date of their construction;

TABLE 3.—STATISTICS OF SHIPPING AND TRADE IN THE PORT OF LONDON

Year	Total net registered tons. Arrivals and departures	Total goods				Shipping in P.L.A. Docks. Arrivals and departures		Trade in P.L.A. Docks. Goods passed over quays		
		Imports : tons	Exports : tons	Transhipments : tons	Total : tons	No. of vessels	Net registered tons	Imports : tons	Exports : tons	Total : tons
1910	39,473,001	†	†	†	†	18,704	21,248,610	1,995,752	640,869	2,636,621
1928	53,141,203	†	†	†	†	19,030	31,088,782	2,344,367	701,124	3,045,491
1938	62,949,744	34,837,356	6,072,680	3,733,608	44,643,644	18,702	32,641,606	2,450,494	771,546	3,222,040
1946	31,017,310	‡	‡	‡	‡	10,672	18,669,288	1,621,315	1,277,732	2,899,047
1952	57,692,297	38,292,417	7,628,410	3,272,690	49,193,517	14,748	26,792,982	2,185,270	2,195,243	4,380,513

† Accurate statistics of tonnage of goods handled in the Port are not available before 1930
‡ Not available.

in some cases, however, the size of shipping has eventually outstripped that of the entrances.

The Upper Entrance at Tilbury, although built in 1928, still provides a good margin, especially in length, over the largest present-day vessels of the P. & O. and Orient Lines, and the King George V Entrance, now 30 years old, has on one occasion accommodated the present R.M.S. "Mauretania," which was built to conform with the dimensions of this entrance; the next largest ship which regularly docks at this entrance is the "Dominion Monarch" of the Shaw, Savill and Albion Line, for which there is a good margin.

Analyses of shipping at the Royal Docks and at the West India Docks, made a few years ago, have shown that the numbers of vessels of the largest sizes are small by comparison with the total and that a second entrance of more moderate dimensions than the largest is a very valuable asset at a dock system, since it enables more shipping to be docked on a tide and conserves impounded water. For instance, it was found that, from consideration of size alone, the Gallions Lower Entrance of the Royal Albert Dock could have locked as much as two-thirds of the number of ships using the Royal Docks and a somewhat similar proportion could have been locked at the Blackwall Entrance of the West India Docks; it is seldom if ever possible economically to justify the provision of two entrances of the largest size at a dock system, but on the other hand it is of the utmost importance that the dimensions of such entrances should be determined from a careful and generous estimate of the sizes of the largest classes of ships that will be built in the future for the trade of the dock.

Once built, these structures can seldom be altered, except at very great expense, and the additional cost of providing a good margin is relatively not great. Past history shows that, in every case, planning in this way has been fully justified. The cost of ships and the value of the cargoes they carry are today so great that the capital cost of adequate port works, while very heavy, is not unduly large by comparison and may be fully justified in the national interest by the economies that the larger ship effects by reduction in running and transportation costs.

An important matter in the determination of the dimensions of entrances is the relationship of beam to length of ships and this has altered considerably during the past 50 years. Existing port works, both at home and abroad, have often imposed limitations upon the increase in the size of ships, especially of length, by reason of locks and berths available, and of draught by reason of the depth of channels, which often are difficult and costly to deepen.

The only freedom available to the shipbuilder, therefore, has been in beam; this may be related to length by the formula:

$$\text{Beam} = \frac{\text{length}}{10} + C$$

The value of the term C was about 3 in 1900 but has steadily increased and is now sometimes as high as 19 and, I understand, may even increase to about 24 in the future.

The question of " the depth of water to be provided in sea ports, their entrances and berths in relation to the present trend in shipbuilding as regards dimensions and speeds of large ocean-going liners, cargo ships and tankers " is at present being studied by the Permanent International Association of Navigation Congresses, on the British National Committee of which the Institution is represented.

When formulating his proposals for the improvement of the Port Authority's Docks in 1910, Sir Frederick Palmer had these considerations prominently in mind and in his comments expressed his only fear, which was that he might repeat the former mistake of building certain entrances too short for future requirements. He considered that such mistakes had repeatedly hampered shipowners in the size of vessels necessary for their trade and could involve very heavy expenditure in reconstruction which could not give the same advantageous results as the original provision of adequate large entrances. His foresight and that of the Port Authority in building the magnificent entrances he designed have been amply justified, although it must be mentioned that his original proposals for length and width were modified.

ACKNOWLEDGEMENTS

In conclusion, I wish to acknowledge the ready help given by the General Manager of the Port of London Authority, the staff of the Chief Engineer, and particularly the enthusiastic research of the Divisional Engineer, New Works.

Mr A. S. Quartermaine, Past-President, moved

> " That the best thanks of the Institution be accorded to the President for his Address and that he be asked to permit it to be printed in the Proceedings of the Institution."

Although, said Mr Quartermaine, the members needed no persuasion from him to give their wholehearted support to the motion, he felt that he must comment upon the outstanding Address which the President had delivered. The wealth of reliable detail included in the Address, and the manner in which the most important things had been compressed into it, gave some indication of the care and skill displayed in its preparation.

He knew Mr Shepherd-Barron well enough to say that he would follow tradition and give his consent to the Address being printed in the Proceedings, and he could assure him that it would go into the archives as a most

valuable historical record of the evolution and development of the famous Port of London.

The President had related an intriguing story of the progress over the past two centuries—a story of changes in depths and dimensions and in design and methods of construction of locks, docks, and warehouses, and even of the advent of a railway service.

There was a rather fascinating sound about the early names of the docks—Canada, Quebec, Greenland, Russia, and Norway Docks and, of course, East and West India Docks. They were an indication of Britain's growing maritime trade. Mr Quartermaine said he had always understood that the development of the West India Dock was largely attributable to the increased imports of rum and sugar, but that must have been in the days before central heating!

Mr Shepherd-Barron was one of a long line of eminent men who had been associated with the docks of London and had mentioned the Institution's founder, H. R. Palmer and the first President, Thomas Telford. But there were many other Past-Presidents including Sir John Rennie, James Rendel, G. P. Bidder, Sir John Hawkshaw, Sir John Wolfe Barry—who had formed a very good working party to build those magnificent docks. After the formation of the Port of London Authority, the Institution had had as Presidents the Authority's first Chief Engineer, Sir Frederick Palmer, and later Sir Cyril Kirkpatrick; Mr Binns had been President-Elect, and now Mr Shepherd-Barron was President. That was an indication of the magnitude and importance of the work, of which the Institution did not need to be reminded but on which its members could congratulate themselves.

Mr. Shepherd-Barron had gone to the Port of London Authority as Chief Engineer about fifteen years ago following extensive and responsible dock work abroad. After a long and honourable career as Chief Engineer with the Authority, he now became the Institution's President. He would be a very worthy successor to those other Past-presidents associated with the London Docks.

The President was to be congratulated heartily on his Address and for the masterly manner in which he had opened his innings.

Mr J. H. Jellett, seconding the motion, said that the epic nature of the story to which they had listened in the Address could almost be called an Iliad which had tonight found a very eminent Homer in the person of their new President.

The President's frequent references to the sterling qualities of the Thames ballast, which played so large a part in the works described in the Address, recalled a small incident of many years ago when, as an assistant engineer in Chatham Dockyard, Mr Jellett had been concerned in the demolition of a large number of concrete blocks taken off a German floating dock. The demolitions had not proceeded very far when the stalwart Kentish ganger, who was in charge of the work, asserted that

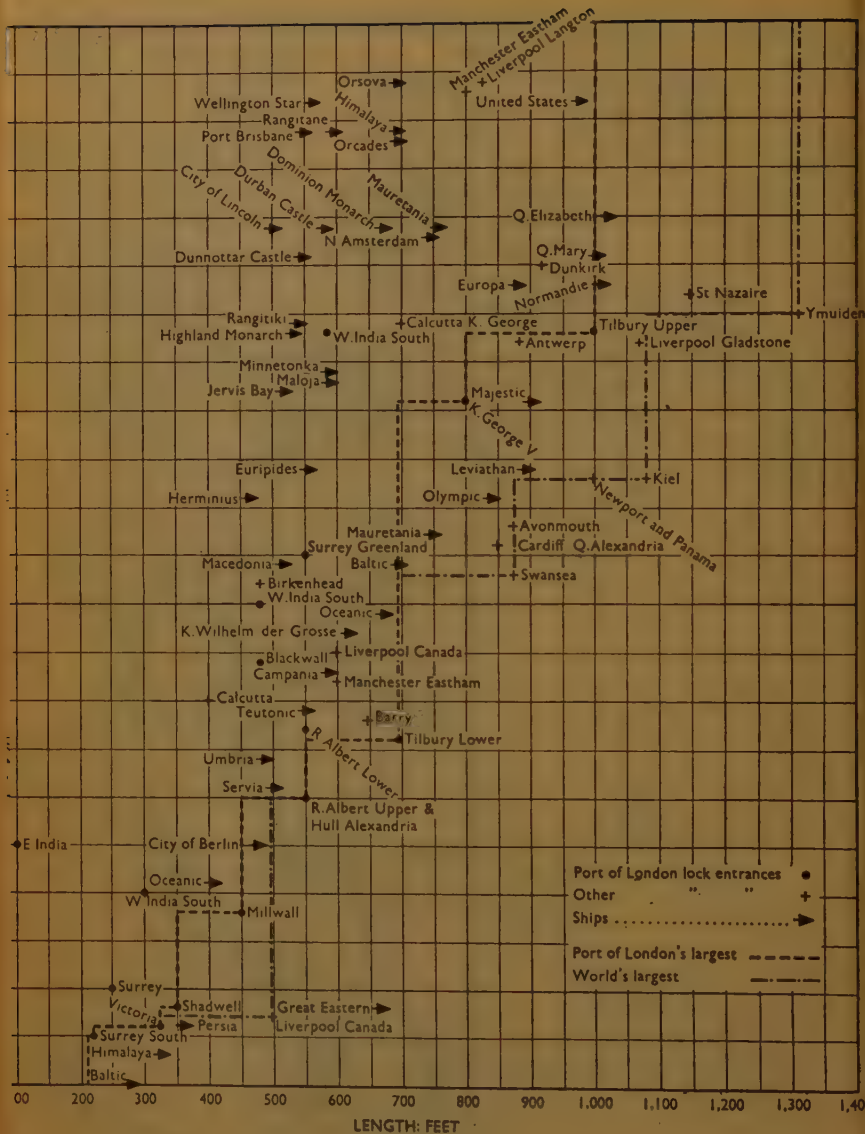
the blocks were not made of concrete. "They are not," he had said, "made with Thames ballast." No persuasion, supported even by references to textbook definitions, had been adequate to shift that man from his view that to make concrete worthy of the name it was necessary to have Thames ballast. A proper territorial pride could be a very adequate foundation for great works and projects of the kind that the President had described.

The motion was carried with acclamation.

The President, in acknowledgement, expressed his thanks to everyone for listening so patiently, and said he would be pleased for his Address to be printed in the Proceedings.

CUTLER ST. WAREHOUSES
COMMERCIAL RD. WAREHOUSE
LONDON DOCKS
EAST INDIA DOCKS
WEST INDIA DOCKS
IMPORT DOCK
EXPORT DOCK
SOUTH DOCK
MILLWALL DOCKS
SURREY COMMERCIAL DOCKS
ROYAL VICTORIA & ALBERT & KING GEORGE V. DOCKS
ROYAL VICTORIA DOCK
ROYAL ALBERT DOCK
KING GEORGE V. DOCK
SILVERTOWN
NORTH WOOLWICH
RIVER THAMES
GRAND SURREY CANAL
PORT OF LONDON AUTHORITY.
PLAN OF THE DOCKS
Scales

TILBURY DOCKS
TILBURY
GRAVESEND
NORTHFLEET
RIVER THAMES



P DIMENSIONS SINCE 1850

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For a Paper presented for discussion at an Ordinary Meeting :—

(1) A Telford Premium to H. D. Morgan, M.Sc.(Eng.), M.I.C.E., P. A. Scott, B.Sc., M.I.C.E., R. J. C. Walton, M.I.C.E., and R. H. Falkiner, B.A., B.A.I., M.I.C.E., jointly, for their Paper on " The Claerwen Dam."

For Papers presented at Meetings of the Engineering Divisions :—

STRUCTURAL AND BUILDING ENGINEERING DIVISION

(1) A Telford Premium to D. H. New, B.Sc.(Eng.), M.I.C.E., for his Paper on " The Design and Construction of the British European Airways Hangars at London Airport, with Particular Reference to Prestressed Concrete."

(2) The Coopers Hill War Memorial Prize to O. N. Arup, C.B.E., M.Sc., M.I.C.E., and R. S. Jenkins, B.Sc.(Eng.), A.M.I.C.E., jointly, for their Paper on " The Design of a Reinforced-Concrete Factory at Brynmawr, South Wales."

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(5) A Telford Premium to E. Windle Taylor, M.A., M.D., D.P.H., M.R.C.S., L.R.C.P., for his Paper on " Bacteria in Relation to Water."

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AIRPORT ENGINEERING DIVISION

(7) A Telford Premium to D. J. Maclean, B.Sc., and P. J. M. Robinson, B.Sc.(Eng.), A.M.I.C.E., jointly, for their Paper on " Methods of Soil Stabilization and their Application to the Construction to Airfield Pavements."

HYDRAULICS ENGINEERING DIVISION

(8) A Telford Premium to C. H. Dobbie, B.Sc.(Eng.), M.I.C.E., and P. O. Wolf, B.Sc. (Eng.), A.M.I.C.E., jointly, for their Paper on " The Lynmouth Flood of August 1952."

Graduates and Students Papers

For Papers read before Local Associations and the Association of London Students :—

(1) A Miller Prize to J. A. S. Watson, Stud.I.C.E., for his Paper on " Summer Work on the Daer Water Scheme Pipelines " (Edinburgh and East of Scotland Association).

(2) A Miller Prize to Kenneth Mackenzie, B.Sc., Grad.I.C.E., for his Paper on " The Lining of Errochty High Pressure Tunnel " (Edinburgh and East of Scotland Association).

(3) A Miller Prize to R. M. Birse, B.Sc., Grad.I.C.E., for his Paper on "Engineering Education" (Edinburgh and East of Scotland Association).
(4) A Miller Prize to M. F. Maggs (former Stud.I.C.E.) for his Paper on "Road Traffic Engineering with Reference to Some Recent Bristol Surveys" (South-Western Association).
(5) A Miller Prize to Douglas Collier, Stud.I.C.E., for his Paper on "Investigations into the Stability of a Masonry Screen Wall" (Edinburgh and East of Scotland Association).
(6) A Miller Prize to M. E. McKee, Grad.I.C.E., for his Paper on "Design of Proposed New Sludge Jetty for Belfast Corporation" (Northern Ireland Association).
(7) A Miller Prize to M. J. Hill, Stud.I.C.E., for his Paper on "First Stage Roadworks of a New Approach Road to Swansea" (South Wales and Monmouthshire Association).

The Institution Medal and Premium (London)

Awarded on the result of an annual competition between undergraduates of the University of London.

To John Geoffrey Measor, Stud.I.C.E., of City and Guilds College, who read a Paper entitled "A Review of Site Consolidation and Drainage by Groundwater Lowering and Other Methods."

ELECTION OF ASSOCIATE MEMBERS

The Council, at their meeting on the 22nd September, 1953, in accordance with By-law 14, declared that the undermentioned had been duly elected as Associate Members.

Home

Bell, Peter Dunbar, B.Sc.Tech. (*Manchester*), Stud.I.C.E.
Brockhurst, Robert Gerald Hubert.
Campbell, Dermont McMurchy, B.Sc. (*Glasgow*), Stud.I.C.E.
Chisholm, Alan, B.Sc. (*Durham*), Stud.I.C.E.
Connell, Ivan Llewellyn, B.A., B.A.I. (*Dublin*), Grad.I.C.E.
Dymock, Thomas, B.Sc. (*Edinburgh*), Stud.I.C.E.
Eelbeck, Alan, Stud.I.C.E.
Feneron, Stanley Graham Stuart, B.Sc.(Eng.) (*London*), Grad.I.C.E.
Foulsham, Michael Sidney, M.A. (*Cantab.*), Stud.I.C.E.
Gordon, Angus Young, B.Sc. (*Glasgow*).
Haigh, Geoffrey, Stud.I.C.E.
Hyde, Roger Bryan, M.A. (*Cantab.*), Stud.I.C.E.
James, John George William, B.A. (*Cantab.*).
Leafe, Frederick Edwin, Grad.I.C.E.
McNaught, John Morton, Stud.I.C.E.
Parsons, Maurice Alwyn, Grad.I.C.E.
Radford, George Graham, B.A. (*Cantab.*).
Senior, Alan Paul.
Swann, Peter Colin, B.Sc.Tech. (*Manchester*), Stud.I.C.E.
Telford, Peter Taylor, B.Sc. (*St Andrews*), Stud.I.C.E.
Toogood, Clarence Aubrey, B.Sc. (*Birmingham*).

DEATHS

It is with deep regret that intimation of the following deaths has been received.

Members

Alec Richard Chesters Ball, B.Sc. (E. 1935, T. 1946).
Alfred Brown Ernest Blackburn, C.B.E. (E. 1901, T. 1920).
Philip Piggott Brown (E. 1907, T. 1937).
Joseph MacLeod Carey, O.B.E. (E. 1935).
William Muirhead (E. 1896, T. 1904).
James Davis Pearson, B.A., B.E. (E. 1903, T. 1913).
Salman Budrudin Tyabji (E. 1912, T. 1943).

Associate Members

Stephen Augustus Bennett (E. 1915).
Lionel Calisch (E. 1908).
Humfrey Alleyne Carte (E. 1894).
Ganapati Iyer Chellam, B.E. (E. 1938).
Professor Purushottam Ganesh Dani, B.Sc.(Eng.) (E. 1910).
Joseph Patrick Lumley, B.E. (E. 1921).
James Ainsworth Settle (E. 1896).
Herbert Charles Palairet Woolmer (E. 1897).

Student

Peter Robert Johnston Kirkpatrick (A. 1945).

Paper No. 5939

"The Elastic Stability of Straight I-beams subjected to Complex Loads"

by

Philip Michael Worthington, B.Sc.(Eng.), Grad.I.C.E.

(*Ordered by the Council to be published with written discussion*) †

SYNOPSIS

The Paper describes a method of checking whether a long straight member of I-section, subjected to any load system, is stable. Only beams within the elastic range are treated. The method is to resolve the problem into a number of simpler problems, analyse each of them separately, and then superpose the results to give an approximate answer to the original case.

The analyses of most of the simple cases have been published before and are well known. In such cases only the results and references to the original works are given here. The theory of superposition of the simple cases is discussed and some general approximate formulae are obtained.

An Appendix is given which may be useful in the design office. In it are tabulated the results of many simple cases, together with the appropriate formulae.

Introduction

Many Papers have been written during the past 50 years on the theory of the elastic stability of beams, and many special cases of beams having certain end conditions and load systems have been solved ; but because of the difficulty of solving complex cases, it has not been usual for the designer to use this work to check his beams for stability. Instead, simplified formulae are used such as that given in British Standard 449 : 1948, paragraph 18. These formulae are generally over-simplified and likely to lead to large errors either on the safe or unsafe side. For example, the formula given in the British Standard has the following weaknesses :—

(1) It does not take into account whether the loads are applied on the top flange of the beam, as in a crane girder, or on the central axis or bottom flange.

† Correspondence on this Paper should be received at the Institution by the 15th May, 1954, and will be published in Part I of the Proceedings. Contributions should be limited to about 1,200 words.—Sec. I.C.E.

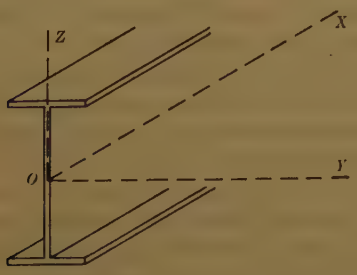
(2) It does not differentiate between bending-moment diagrams of different shapes.
(3) It does not differentiate between beams having different degrees of fixity at the ends.
(4) It does not take into account the increased torsional rigidity given by the lateral bending stiffness of the flanges.
(5) It makes some other approximations. For example, it assumes that the torsional stiffness of the beam can be expressed in terms of the radius of gyration of the beam section perpendicular to the plane of bending.

In this Paper a method is given for the checking of beams for stability, which is much more accurate but still simple enough to be used in a design office. When a beam is subjected to a complex system of loads, the method is to resolve the problem into several more simple hypothetical cases, then find the critical load of each hypothetical case by reference to Tables and finally to superpose the results.

Stiffness Constants of the Beam

Throughout this Paper the axis of X is defined as the undeflected position of the torsional axis of the beam, and the mutually perpendicular axes of Y and Z as shown in *Fig. 1.*

Fig. 1



The flexural rigidity of the beam about the Y- and Z-axes will be denoted by β_1 and β_2 respectively. By flexural rigidity is meant the moment required to produce unit curvature of the beam at any section. Thus

$$\left.\begin{aligned}\beta_1 &= EI_1\\ \text{and}\qquad \beta_2 &= EI_2\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (1)$$

where E denotes Young's modulus of elasticity and I_1 and I_2 the second moments of area of the section about the axes of β_1 and β_2 respectively.

The equation of twist of an I-beam may be written

$$T = \gamma \frac{d\theta}{dx} - \alpha \frac{d^3\theta}{dx^3} \quad \ldots\ldots\ldots \quad (2)^*$$

where T denotes the torque and θ the angle through which a section has rotated from its natural position; γ denotes the St Venant's torsional stiffness and α denotes Wagner's torsional constant. An approximate formula for γ for a beam consisting of a thin web and two thin flanges is:

$$\gamma = \frac{N}{3}(Dt_w^3 + 2Bt_f^3) \quad \ldots\ldots\ldots \quad (3)\dagger$$

where D and B denote the web depth and flange breadth respectively, and t_w and t_f the web and flange thicknesses respectively. N denotes the shear modulus of elasticity.

An approximate formula for α is

$$\alpha = \frac{\beta_2 D^2}{4} \quad \ldots\ldots\ldots \quad (4)\ddagger$$

If there is an axial thrust P in the beam, equation (2) is extended to:

$$\left(\gamma - \frac{P}{r^2}\right)\frac{d\theta}{dx} - \alpha \frac{d^3\theta}{dx^3} = 0 \quad \ldots\ldots\ldots \quad (5)\S$$

where r denotes the polar radius of gyration of the section of the beam.

Definition of Potential Couples

Consider a concentrated load W acting in the direction of the Z-axis and applied at a point above or below the central axis of the beam, as shown in *Fig. 2(a)*. This load can be resolved into a load of equal magnitude applied at the central axis of the beam as shown in *Fig. 2(b)* and a pair of equal and opposite loads as shown in *Fig. 2(c)*. The single load applied at the central axis will be termed the transferred load, and the product Wh the potential couple, h being the distance between the points of application of the pair of equal and opposite loads. A potential couple puts the beam in a state of torque only when it is twisted from its normal position about the X-axis. In *Figs 2* the beam is shown in a slightly twisted state.

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* Equation (2) was derived for the I-beam by S. Timoshenko ("Theory of Elastic Stability," McGraw-Hill, p. 256). It was derived previously for the general open section by H. Wagner ("Torsion and Buckling of Open Sections," 25th Anniv. Pubns Tech. Hockschule, Danzig).

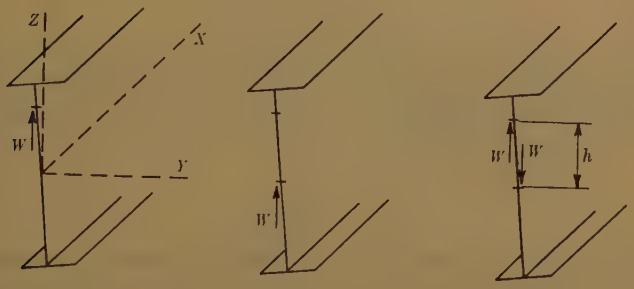
† The derivation of formula (3) is given by S. Timoshenko in "Strength of Materials," vol. I, MacMillan, p. 78.

‡ S. Timoshenko, "Theory of Elastic Stability," p. 256 *et seq.*

§ S. Goodier, "Torsional and Flexural Buckling of Bars of Thin Walled Open Section Under Compressive and Bending Loads," J. Appl. Mech., U.S.A., 1942. Formula (5) was first published by H. Wagner.

Figs 2

(*a*) (*b*) (*c*)



The Elementary Component Load Systems

Any system of loads applied to a beam can be resolved into the following five component systems which will be called the elementary component systems :—

(1) Loads in the direction of the X-axis applied at points on the central axis of the beam.
(2) Torques about the X-axis.
(3) Potential couples acting in planes normal to the X-axis.
(4) Moments about the Y-axis and loads in the direction of the Z-axis applied at points on the central axis of the beam.
(5) Moments about the Z-axis and loads in the direction of the Y-axis applied at points on the central axis of the beam.

The stability of the beam when subjected to each elementary component system separately will be considered first, and will be followed by a discussion on the method of superposing the results so that the stability of the beam can be checked when it is subjected to the resultant system.

Axial Loads

A beam subjected to a system of axial loads can, apart from local instability, become unstable in one of three ways. It can bend about the Y-axis, bend about the Z-axis, or twist about the X-axis. If the critical magnitude of the thrust P which will just cause the beam to become unstable about these three axes be denoted by P_{yc}, P_{zc}, and $P_{\theta c}$ respectively, then :

$$P_{yc} = p_{yc}\frac{\beta_2}{l^2} \quad \ldots\ldots\ldots\ldots \quad (6a)^*$$

* Formulae (6*a*) and (6*b*) were first published by L. Euler (*Histoire de l'Académie*, vol. 13, pp. 252–282, Berlin, 1757).

$$P_{zc} = p_{zc}\frac{\beta_1}{l^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6b)^*$$

and

$$P_{\theta c} = \frac{\gamma}{r^2} - p_\theta \frac{a}{l^2 r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6c)\dagger$$

where p_y, p_z, and p_θ are constants which depend only on the end conditions and loading conditions of the beam. Values of these constants for several special cases are given in the Appendix.

Torques

When a beam is subjected to a torque or a system of torques and becomes unstable, it bends about both axes of flexure and takes the form of a helix. If T_c denotes the critical value of the system of torques then :

$$T_c = \pm t\frac{\beta_1\beta_2}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)\ddagger$$

where t is a constant depending only on the end conditions of the beam and the distribution of the applied torques. Values of this constant for a few special cases are given in the Appendix.

Potential Couples

For an example consider a cantilever of length l with a concentrated potential couple Q, as defined in the section entitled " The Elementary Component Load Systems," applied at the free end. If θ_0 denotes the angle of twist at the free end when the beam is in neutral equilibrium, the torque in the cantilever is $Q\theta_0$ along the whole length. Hence, from equation (2) :

$$\gamma\frac{d\theta}{dx} - a\frac{d^3\theta}{dx^3} = Q\theta_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The solution of this equation may be written :

$$\theta = a_1 \sinh \mu x + a_2 \cosh \mu x + a_3 + \frac{Q\theta_0}{\gamma}x \quad . \quad . \quad . \quad (9)$$

where

$$\mu^2 = \frac{\gamma}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Taking the origin at the free end of the cantilever, the end conditions are such that $\theta = \theta_0$ and $\frac{d^2\theta}{dx^2} = 0$ at $x = 0$ and that $\theta = 0$ and $\frac{d\theta}{dx} = 0$ at $x = l$.

* See footnote on p. 49.

† Formula (6c) follows from equation (5).

‡ The special case of formula (7), when $\beta_1 = \beta_2$ was given by A. G. Greenhill (" On the Strength of Shafting when Exposed both to Torsion and End Thrust," Proc. Instn Mech. Engrs, vol. 34, 1883).

Putting these conditions in equation (9) and eliminating the constants a_1, a_2, and a_3 it can be deduced that :

$$Q = \frac{\mu\gamma}{\tanh \mu l - \mu l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

This equation gives the value of Q at which the cantilever is in neutral equilibrium. The cantilever fails by just twisting about the X-axis. In the special case of a beam in which $\alpha = 0$, the value of Q is found by writing equation (11) in the form :

$$Q = \frac{\gamma}{\frac{1}{\mu}\tanh \mu l - \mu l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and then making μ large. Thus :

$$Q = -\frac{\gamma}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

In the special case when $\gamma = 0$, the value of Q can be found by expanding $\tanh \mu l$ in ascending powers of μl. Thus equation (11) becomes :

$$Q = \frac{\mu\gamma}{(\mu l - \frac{1}{3}\mu^3 l^3 + \ldots\ldots) - \mu l}$$

If now γ is expressed in terms of μ and α, by means of equation (10), and μ is made small, then :

$$Q = -\frac{3\alpha}{l^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Consideration may now be given to the general case of a beam having any given end conditions and subjected to any given system of potential couples. If, first, it be assumed that $\alpha = 0$ and the critical magnitude of the system of potential couples be denoted by $Q_{\gamma c}$, then :

$$Q_{\gamma c} = q_\gamma \frac{\gamma}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where q_γ is a constant depending only on the end conditions of the beam and the distribution of the potential couples. Similarly, if it be assumed that $\gamma = 0$ but α has its true value, then, denoting the critical magnitude by $Q_{\alpha c}$:

$$Q_{\alpha c} = q_\alpha \frac{\alpha}{l^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Several values of the constants q_γ and q_α have been worked out and are given in the Appendix. The method of superposing the values of $Q_{\gamma c}$ and $Q_{\alpha c}$ to give the critical value of Q when neither α nor γ are supposed zero is discussed later in the Paper.

Moments about the Y-axis and Loads in the Direction of the Z-axis

Suppose the beam is subjected only to a system of moments about the Y-axis and loads in the direction of the Z-axis applied at points on the central axis of the beam. The general case when neither α nor γ are zero is usually difficult to solve. It is proposed, therefore, to deal only with the two hypothetical cases when $\alpha = 0$ and $\gamma = 0$ separately as in the case of potential couples. The superposition of the hypothetical cases will be described later.

In the hypothetical case when $\alpha = 0$, let $M_{\gamma c}'$ denote the critical magnitude of the highest point of the bending-moment diagram; then:

$$M_{\gamma c}' = \pm \frac{m_\gamma}{l}\sqrt{\frac{\beta_1\beta_2\gamma}{\beta_1 - \beta_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)^*$$

Similarly, if $M_{\alpha c}'$ denotes the critical magnitude in the hypothetical case when $\gamma = 0$, then:

$$M_{\alpha c}' = \pm \frac{m_\alpha}{l^3}\sqrt{\frac{\beta_1\beta_2\alpha}{\beta_1 - \beta_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)^\dagger$$

The constants m_γ and m_α depend only upon the end conditions of the beam and the shape of the bending-moment diagram. Values for some special cases are given in the Appendix.

Moments about the Z-axis and Loads in the Direction of the Y-axis

Now suppose the beam to be subjected only to a system of moments about the Z-axis and loads in the direction of the Y-axis applied at points on the central axis of the beam. By analogy with equations (17) and (18), the critical magnitudes in the two hypothetical cases are given by:

$$M_{\gamma c}'' = \pm \frac{m_\gamma}{l}\sqrt{\frac{\beta_1\beta_2\gamma}{\beta_2 - \beta_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

and

$$M_{\alpha c}'' = \pm \frac{m_\alpha}{l^3}\sqrt{\frac{\beta_1\beta_2\alpha}{\beta_2 - \beta_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Both $M_{\gamma c}''$ and $M_{\alpha c}''$ are imaginary quantities since β_2 is less than β_1. A beam subjected to such a system of loads cannot, therefore, become unstable. When discussing the superposition of the elementary component load systems it will be found that such a system of loads has a stabilizing effect on the beam.

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* Formula (17) was first published by A. G. Mitchell (*Philosophical Magazine*, vol. 48, 1899).

† S. Timoshenko, "The Theory of Elastic Stability," p. 259.

Fundamental Modes

In the section entitled " The Elementary Component Load Systems " the five " elementary component load systems " were defined. The " critical magnitude " of an elementary load system is defined as the least magnitude which will put the beam into neutral equilibrium. The mode of deflexion in which the beam is then in neutral equilibrium is known as the " fundamental mode."

In the section entitled " Axial Loads " it was explained that if a system of axial loads is applied to a beam there are three independent fundamental modes, one consisting of deflexions in the direction of the Y-axis, one consisting of deflexions in the direction of the Z-axis, and one consisting of twist about the X-axis. In general these modes have different critical loads and no two of the modes occur together. Each of these modes is called a " one-dimensional mode."

A system of torques about the X-axis has a fundamental mode which can be resolved into two components, one consisting of deflexions in the direction of the Y-axis and the other of deflexions in the direction of the Z-axis. This type of mode is therefore called a " two-dimensional mode " and the resolved parts are the " principal components " of the mode.

Table 1 gives a summary of the types of fundamental mode for all of the five elementary load systems.

Table 1

Type of elementary load system	Type of fundamental mode	Dimensions of mode
(1) Axial loads	One-dimensional	y, z, or θ
(2) Torques	Two-dimensional	y and z
(3) Potential couples	One-dimensional	θ
(4) Moments about the Y-axis and loads in the direction of the Z-axis	Two-dimensional	y and θ
(5) Moments about the Z-axis and loads in the direction of the Y-axis	Two-dimensional	z and θ

Energy Principles

The following three principles concerning the energies of the beam and applied loads will be used later when deriving the formulae for superposing the elementary load systems.

The Principle of Constant Energy

If the beam is in neutral equilibrium under the action of a system of loads, the total energy of the beam and loads is independent of the amplitude of the mode of deflexion. It follows that as the amplitude is

increased, the gain in strain energy of the beam is equal to the loss in potential energy of the system of applied loads.

The Principle of Minimum Strain Energy

If a beam is in equilibrium under the action of a system of loads, then for any fixed potential energy of the applied loads the mode of deflexion is such that the strain energy is minimum. For if this is not the case the mode can be altered slightly in such a way that the strain energy of the beam is decreased while the potential energy remains the same. But such a condition is not consistent with the beam being in equilibrium.

The principle can be stated symbolically in the following manner. Let the mode of deflexion be given by the equations :

$$\left.\begin{aligned} y &= a_0 + a_1 x + a_2 x^2 + \ldots\ldots, \\ z &= b_0 + b_1 x + b_2 x^2 + \ldots\ldots, \\ \theta &= c_0 + c_1 x + c_2 x^2 + \ldots\ldots, \end{aligned}\right\} \quad \ldots \quad (21)$$

the constants $a_0, a_1, \ldots\ldots, b_0, b_1, \ldots\ldots, c_0, c_1, \ldots\ldots$ satisfying all the end conditions and other restraints. Then the principle of minimum strain energy states that :

$$\left.\begin{aligned} \frac{\partial E}{\partial a_0} &= 0, \quad \frac{\partial E}{\partial a_1} = 0, \ldots\ldots, \\ \frac{\partial E}{\partial b_0} &= 0, \quad \frac{\partial E}{\partial b_1} = 0, \ldots\ldots, \\ \frac{\partial E}{\partial c_0} &= 0, \quad \frac{\partial E}{\partial c_1} = 0, \ldots\ldots, \end{aligned}\right\} \quad \ldots\ldots \quad (22)$$

Principle of Maximum Loss of Potential Energy

Conversely it can be stated that if a beam is in equilibrium under the action of a system of loads, then for any fixed strain energy of the beam the mode of deflexion is such that the loss in potential energy of the loads is maximum.

A Property of Two-Dimensional Modes of Elementary Load Systems

Before deriving formulae for the superposition of the elementary component load systems, it will be found convenient to state a property of two-dimensional modes. This is that, in a beam subjected to an elementary load system having a two-dimensional fundamental mode, the strain energies of the two principal components of the mode are equal.

To prove this, first consider a beam subjected to a load system of the fourth elementary type. The fundamental mode consists of flexure about the Y-axis and twist about the X-axis. The deflexion of the beam in the

direction of the Z-axis due to these two principal components of the mode is given by:

$$\frac{dz}{dx} = \theta \frac{dy}{dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

It is not possible to integrate this equation directly because θ is a function of x. Nevertheless the equation shows that, at any section of the beam, z is proportional to the product of the amplitudes of the two principal components of the modes. The loss in potential energy of the applied loads is therefore also proportional to the same product.

If E_y and E_θ denote the strain energies of the principal components of the mode, the amplitudes are proportional to $\sqrt{E_y}$ and $\sqrt{E_\theta}$ respectively. Hence, if V is the loss of potential energy of the elementary load system it can be stated that:

$$V = K\sqrt{E_y E_\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where

$$E_y + E_\theta = E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

E denoting the total strain energy and K being a constant. Expressing $E_y E_\theta$ in terms of E and $\frac{E_y}{E_\theta}$, equation (24) may be written in the form:

$$\frac{E}{V} = \frac{1}{K}\left(2 + \frac{E_y}{E_\theta} + \frac{E_\theta}{E_y}\right)$$

Keeping V constant and differentiating with respect to $\frac{E_\theta}{E_\gamma}$ gives:

$$\frac{1}{V}\frac{dE}{d\left(\frac{E}{E_y}\right)} = \frac{1}{K}\left\{-\left(\frac{E_\theta}{E_y}\right)^2 + 1\right\}$$

But, by the principle of minimum strain energy, $\dfrac{dE}{d\left(\frac{E_\theta}{E_y}\right)} = 0$

Therefore $E_\theta = E_y$ or, from equation (25):

$$E_\theta = \frac{E}{2}; \text{ and } E_y = \frac{E}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

This property can be proved in a similar way for the other two elementary load systems having two-dimensional characteristic modes.

Superposition of the Elementary Component Load Systems

Let P be a system of axial thrusts, T a system of torques, Q a system of potential couples, M' a system of moments about the Y-axis and loads in the direction of the Z-axis, and M'' a system of moments about the Z-axis and loads in the direction of the Y-axis. Also let P_{yc}, P_{zc}, and $P_{\theta c}$ be the

critical magnitudes of P for modes in the directions of y, z, and θ respectively and let T_c, Q_c, M_c', and M_c'' be the critical magnitudes of T, Q, M', and M'' respectively. Then if all the load systems act together, the beam is in neutral equilibrium when :

$$\left(\frac{M''}{M_c''}\right)^2\left(1-\frac{P}{P_{yc}}\right)+\left(\frac{M'}{M_c'}\right)^2\left(1-\frac{P}{P_{zc}}\right)+\left(\frac{T}{T_c}\right)^2\left(1-\frac{P}{P_{\theta c}}-\frac{Q}{Q_c}\right)$$
$$+2\sqrt{-1}\frac{M'M''T}{M_c'M_c''T_c}-\left(1-\frac{P}{P_{yc}}\right)\left(1-\frac{P}{P_{zc}}\right)\left(1-\frac{P}{P_{\theta c}}-\frac{Q}{Q_c}\right)\geqslant 0 \quad . \quad . \quad . \quad (27)$$

To prove this formula it is first assumed that in each of the dimensions y, z, and θ, the principal components of the fundamental modes of all the elementary load systems are similar. By the principle of maximum loss of potential energy it can be seen that the principal components of the fundamental modes of the elementary load systems are such that each of the elementary load systems has a maximum loss of potential energy. Hence, these same principal components give the maximum loss of potential energy of all the load systems when acting together. Therefore, by the principle of maximum loss of potential energy, the principal components of the fundamental mode of the combined load system are similar to those of the elementary systems.

When the beam is in neutral equilibrium under the action of the combined load system, let E_y, E_z, and E_θ denote the strain energies of the principal components of the fundamental mode in the directions y, z, and θ respectively, and let E denote the total strain energy. Then :

$$E = E_y + E_z + E_\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

The loss in potential energy of the system of axial loads is :

$$\frac{P}{P_{yc}}E_y + \frac{P}{P_{zc}}E_z + \frac{P}{P_{\theta c}}E_\theta$$

The loss in potential energy of the system of torques is :

$$\frac{T}{T_c}\frac{E_y}{E_y'}\frac{E_z}{E_z'}(E_y' + E_z')$$

where E_y' and E_z' are the strain energies corresponding to E_y and E_z when only the system of torques acts. But by equations (27) :

$$E_y' = \tfrac{1}{2}(E_y' + E_z') \quad \text{and} \quad E_z' = \tfrac{1}{2}(E_y' + E_z').$$

Therefore, the loss in potential energy of the system of torques when all the loads act together is $2\frac{T}{T_c}\sqrt{E_yE_z}$. Similar expressions may be written for the loss in potential energy of the other elementary load systems. If V denotes the total loss in potential energy of the combined system of loads, then :

$$V = \frac{P}{P_{yc}} E_y + \frac{P}{P_{zc}} E_z + \frac{P}{P_{\theta c}} E_\theta + 2\frac{T}{T_c}\sqrt{E_y E_z} + \frac{Q}{Q_c} E_\theta + 2\frac{M'}{M_c'}\sqrt{E_y E_\theta} + 2\frac{M''}{M_c''}\sqrt{-E_z E_\theta} \quad . \quad . \quad . \quad . \quad . \quad (29)$$

Using equation (28) to express E_y, E_z, and E_θ in terms of E, $\frac{E_y}{E_\theta}$ and $\frac{E_z}{E_\theta}$, equation (29) may be written:

$$V\left(1 - \frac{E_y}{E_\theta} + \frac{E_z}{E_\theta}\right) = \frac{P_y}{P_{yc}}\frac{E_y}{E_\theta} + \frac{P}{P_{zc}}\frac{E_z}{E_\theta} + \frac{P}{P_{\theta c}} + \frac{Q}{Q_c} + 2\frac{M'}{M_c'}\left(\frac{E_y}{E_\theta}\right)^{\frac{1}{2}} + 2\frac{M''}{M_c''}\left(-\frac{E_z}{E_\theta}\right)^{\frac{1}{2}} + 2\frac{T}{T_c}\left(\frac{E_y}{E_\theta}\right)^{\frac{1}{2}}\left(\frac{E_z}{E_\theta}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Keeping V and $\frac{E_z}{E_\theta}$ constant and differentiating with respect to $\frac{E_y}{E_\theta}$ gives:

$$\frac{V}{E} - \frac{V}{E^2}\frac{\partial E}{\partial\left(\frac{E_y}{E_\theta}\right)}\left(1 + \frac{E_y}{E_\theta} + \frac{E_z}{E_\theta}\right) = \frac{P}{P_{yc}} + \frac{M'}{M_c'}\left(\frac{E_y}{E_\theta}\right)^{-\frac{1}{2}} + \frac{T}{T_c}\left(\frac{E_y}{E_\theta}\right)^{-\frac{1}{2}}\left(\frac{E_z}{E_\theta}\right)^{\frac{1}{2}}$$

Similarly, keeping V and $\frac{E_y}{E_\theta}$ constant and differentiating with respect to $\frac{E_z}{E_\theta}$ gives:

$$\frac{V}{E} - \frac{V}{E^2}\frac{\partial E}{\partial\left(\frac{E_z}{E_\theta}\right)}\left(1 + \frac{E_y}{E_\theta} + \frac{E_z}{E_\theta}\right) = \frac{P}{P_{zc}} + \frac{M''}{M_c''}\left(-\frac{E_z}{E_\theta}\right)^{-\frac{1}{2}} + \frac{T}{T_c}\left(\frac{E_y}{E_\theta}\right)^{\frac{1}{2}}\left(\frac{E_z}{E_\theta}\right)^{-\frac{1}{2}}$$

But, by the principle of minimum strain energy, $\frac{\partial E}{\partial\left(\frac{E_y}{E_\theta}\right)} = 0$

and $\frac{\partial E}{\partial\left(\frac{E_z}{E_\theta}\right)} = 0$, and by the principle of constant energy $V = E$.

Therefore $$\frac{P}{P_{yc}} + \frac{M'}{M_c'}\left(\frac{E_y}{E_\theta}\right)^{-\frac{1}{2}} + \frac{T}{T_c}\left(\frac{E_y}{E_\theta}\right)^{-\frac{1}{2}}\left(\frac{E_z}{E}\right)^{\frac{1}{2}} = 1$$

and $$\frac{P}{P_{zc}} + \frac{M''}{M_c''}\left(-\frac{E_z}{E_\theta}\right)^{-\frac{1}{2}} + \frac{T}{T_c}\left(\frac{E_y}{E_\theta}\right)^{-\frac{1}{2}}\left(\frac{E_z}{E_\theta}\right)^{\frac{1}{2}} = 1$$

Eliminating E_y and E_z from these two equations and equation (30), and again putting $E = V$, gives :

$$\left(\frac{M''}{M_c''}\right)^2\left(1-\frac{P}{P_{yc}}\right)+\left(\frac{M'}{M_c'}\right)^2\left(1-\frac{P}{P_{zc}}\right)+\left(\frac{T}{T_c}\right)^2\left(1-\frac{P}{P_{\theta c}}-\frac{Q}{Q_c}\right)$$
$$+2\sqrt{-1}\,\frac{M'M''T}{M_c'M_c''T_c}-\left(1-\frac{P}{P_{yc}}\right)\left(1-\frac{P}{P_{zc}}\right)\left(1-\frac{P}{P_{\theta c}}-\frac{Q}{Q_c}\right)=0 \quad . \quad . \quad . \quad (31)$$

If in any one of the dimensions y, z, or θ, the components of the elementary load systems are not all similar, it can be shown by the principle of maximum loss of potential energy that the left-hand side of equation (31) is greater than zero.

Superposition of the Hypothetical Cases $\alpha = 0$ and $\gamma = 0$

In the section entitled " Potential Couples " and the two succeeding sections it was found convenient to consider the stability of the beam first in the hypothetical case when $\alpha = 0$ and then in the hypothetical case when $\gamma = 0$. It is now convenient to discuss the methods of superposing these two cases to give the critical load when neither α nor γ are supposed zero.

(*1*) *Potential Couples*

Let Q_c be the critical magnitude of the system of potential couples when neither α nor γ is supposed zero, and let $Q_{\gamma c}$ and $Q_{\alpha c}$ be the critical magnitudes in the hypothetical cases when α and γ are separately taken as zero. Then :

$$Q_c \geqslant Q_{\gamma c} + Q_{\alpha c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

To prove this formula it is first assumed that the fundamental modes in the two hypothetical cases are similar. From the principle of minimum strain energy it follows that, for a given potential energy of the applied load system, the common fundamental mode of the hypothetical cases is that which gives the minimum strain energy in each of the hypothetical cases. Hence, this same mode gives the minimum strain energy when neither α nor γ is supposed zero. Therefore, by the principle of minimum strain energy, this mode is the fundamental mode when neither α nor γ is supposed zero.

When the fundamental mode has a certain amplitude, let E_γ be the strain energy of pure torsion and E_α be the strain energy due to differential flange bending. The total strain energy is then $E_\gamma + E_\alpha$. By the principle of constant energy, the loss in potential energy of $Q_{\gamma c}$ in the hypothetical case when $\alpha = 0$ is E_γ, the loss in potential energy of $Q_{\alpha c}$ in the hypothetical case when $\gamma = 0$ is E_α, and the loss in potential energy of Q_c when neither α nor γ is supposed zero is $E_\gamma + E_\alpha$. Hence, since the deflexion of the applied load system is the same in the three cases :

$$\frac{Q_{\gamma c}}{Q_c} = \frac{E_\gamma}{E_\gamma + E_a} \quad \text{and} \quad \frac{Q_{ac}}{Q_c} = \frac{E_a}{E_\gamma + E_a}$$

erefore : $$Q_c = Q_{\gamma c} + Q_{ac} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

If, in the hypothetical cases when α and γ are separately zero, the ıdamental modes are not similar, it follows from the principle of maxi-ım loss of potential energy that $Q_c > Q_{\gamma c} + Q_{ac}$

Elementary Load Systems of the Fourth and Fifth Kinds

Let M_c be the critical magnitude of an elementary load system having a o-dimensional fundamental mode, one principal component consisting twist about the X-axis and the other consisting of bending about either Y- or Z-axes. Also let $M_{\gamma c}$ and M_{ac} be the critical magnitudes of the d system in the hypothetical cases. Then :

$$M_c^2 \geqslant M_{\gamma c}^2 + M_{ac}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

To prove this formula it is first assumed that the fundamental modes in hypothetical cases have similar principal components. In this case principal components in the real case, when neither α nor γ is supposed o, are similar to those of the hypothetical cases.

In each of the three cases let the amplitude of the mode be such that principal component of bending has an amplitude b, and let the strain rgy of the component be $\frac{1}{2}E$. Then by equations (26), the energy of component of twist in each of the three cases is also $\frac{1}{2}E$. In the real e let the energy of twist consist of E_γ due to the pure torsional stresses d E_a due to differential flange bending. Then :

$$E_\gamma + E_a = \tfrac{1}{2}E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

If c denotes the amplitude of twist in the real case, the amplitude in the pothetical case when $\alpha = 0$ is $c\sqrt{\frac{\frac{1}{2}E}{E_\gamma}}$ and the amplitude in the hypo-tical case when $\gamma = 0$ is $c\sqrt{\frac{\frac{1}{2}E}{E_a}}$. Since the deflexion of the applied d system is proportional to the product of the amplitudes of the prin-al components of the load, the losses in potential energy of the load tem in the real and hypothetical cases are proportional to $M_c bc$, $_c bc\sqrt{\frac{\frac{1}{2}E}{E_\gamma}}$, and $M_{ac} bc\sqrt{\frac{\frac{1}{2}E}{E_a}}$ respectively. But by the principle of con-nt energy, the loss in potential energy is E in each of the three cases. erefore :

$$M_c bc = M_{\gamma c} bc\sqrt{\frac{\frac{1}{2}E}{E_\gamma}} = M_{ac} bc\sqrt{\frac{\frac{1}{2}E}{E_a}} \quad . \quad . \quad . \quad . \quad (36)$$

minating E_γ and E_a from equations (35) and (36) gives :

$$M_c^2 = M_{\gamma c}^2 + M_{ac}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

If the principal components of the fundamental modes are not similar in the two hypothetical cases, it follows from the principle of maximum loss of potential energy that

$$M_c^2 > M_{\gamma c}^2 + M_{ac}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

Application to Design

Substitution for $Q_{\gamma c}$ and Q_{ac} from equations (15) and (16) into formula (32) gives:

$$Q \geqslant q_\gamma \frac{\gamma}{l} + q_a \frac{a}{l^3}$$

Then, using the equation $$Q_c = q_\gamma \frac{\gamma}{l} + q_a \frac{a}{l^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

when checking the stability of a beam, the estimate of Q_c will therefore be either correct or on the safe side.

Substituting for $M_{\gamma c}'$ and M_{ac}' from equations (17) and (18) into formula (38) gives:

$$M_c'^2 \geqslant \left(\frac{\beta_1\beta_2}{\beta_1 - \beta_2}\right)\left(\frac{m_\gamma^2\gamma}{l^2} + \frac{m_a^2 a}{l^4}\right)$$

Then, using the equation $$M_c'^2 = \left(\frac{\beta_1\beta_2}{\beta_2 - \beta_1}\right)\left(\frac{m_\gamma^2\gamma}{l^2} + \frac{m_a^2 a}{l^4}\right) \quad . \quad . \quad (40)$$

when checking the stability of a beam, the estimate of M_c' will therefore be either correct or on the safe side.

Similarly, when using the equation:

$$M_c''^2 = \left(\frac{\beta_1\beta_2}{\beta_2 - \beta_1}\right)\left(\frac{m_\gamma^2\gamma}{l^2} + \frac{m_a^2 a}{l^4}\right) \quad . \quad . \quad . \quad . \quad (41)$$

when checking the stability of a beam, the estimate of M_c'' will be either correct or on the safe side.

Although in most cases equations (39), (40), and (41) give critical magnitudes which are smaller than the actual critical magnitudes, the equations are accurate enough for ordinary design purposes. Only in rare cases is the error greater than 20 per cent.

When the beam is in neutral equilibrium, formula (27) is satisfied. Therefore when the left-hand side of the formula is less than unity the beam is stable. If the beam is being designed with a factor of safety n, the beam is safe if:

$$\left(\frac{nM''}{M_c''}\right)^2\left(1 - \frac{nP}{P_{\gamma c}}\right) + \left(\frac{nM'}{M_c'}\right)^2\left(1 - \frac{nP}{P_{zc}}\right) + \left(\frac{nT}{T_c}\right)^2\left(1 - \frac{nP}{P_{\theta c}} - \frac{nQ}{Q_c}\right)$$
$$+ 2\frac{n^3 M'M''T}{M_c'M_c''T_c} - \left(1 - \frac{nP}{P_{\gamma c}}\right)\left(1 - \frac{nP}{P_{zc}}\right)\left(1 - \frac{nP}{P_{\theta c}} - \frac{nQ}{Q_c}\right) \leqslant 0$$
$$. \quad . \quad . \quad (42)$$

ACKNOWLEDGEMENT

The Author gratefully acknowledges the help of Messrs W. S. Atkins and Partners in computing the coefficients given in the Appendix.

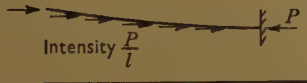
The Paper is accompanied by seven sheets of diagrams, from which the Figures in the text have been prepared, and by the following Appendix.

APPENDIX

To check the stability of a beam the load system is first resolved into its five elementary component systems and an estimate of the critical magnitude of each one obtained by means of the Tables and formulae given below. The beam is then stable if formula (42) is satisfied.

Axial Loads

$$P_{yc} = p\frac{\beta_2}{l^2},\quad P_{zc} = p\frac{\beta_1}{l^2},\quad P_{\theta c} = \frac{\gamma}{r^2} + p\frac{a}{r^2}$$

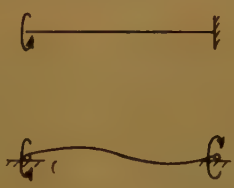
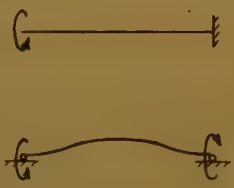
Elevation and plan	End fixity in twist*	p
	Free and encastré	2·47
	Both simple	9·87
	Simple and encastré	20·1
	Both encastré	39·5
	Free and encastré	7·84 †

* An end is said to be " free " in twist when there are no restraints, " simply supported " in twist when it is restrained from rotating but free to warp, and " encastré " in twist when it is restrained from rotating and warping.

† The solution of this case was found by A. G. Greenhill, " Determination of the greatest height consistent with stability that a vertical pole or mast can be made." Proc. Camb. Phil. Soc., vol. 4, p. 65, 1881.

Torques

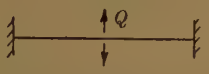
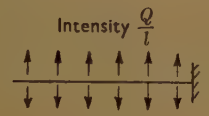
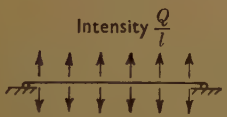
$$T = \frac{t}{l}\sqrt{\beta_1\beta_2}$$

Elevation	Plan	t
		∞
		6·28

* A beam cannot become unstable under a uniform torque unless it is free to take one complete cycle of a cylindrical spiral, the axis of which remains parallel to the X-axis.

Potential couples

$$Q_c = q_\gamma \frac{\gamma}{l} + q_\alpha \frac{a}{l^3}$$

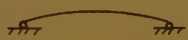
Load system	End fixity in twist*	q_γ	q_α
	Free and encastré	-1	-3
	Both simple	-4	-48
	Both encastré	-4	-192
	Free and encastré	-2·47	-6·09
	Both simple	-9·87	-97·3
	Both encastré	-9·87	-500

* See footnote *, p. 61.

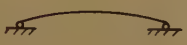
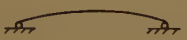
Transverse Moments and Loads

$$M_c'^2 = \left(\frac{\beta_1\beta_2}{\beta_1 - \beta_2}\right)\left(\frac{m_\gamma{}^2\gamma}{l^2} + \frac{m_\alpha{}^2\alpha}{l^4}\right)$$

$$M_c''^2 = -\left(\frac{\beta_1\beta_2}{\beta_1 - \beta_2}\right)\left(\frac{m_\gamma{}^2\gamma}{l^2} + \frac{m_\alpha{}^2\alpha}{l^4}\right)$$

Elevation and B.M. diagram	Plan	End fixity in twist*	m_δ	m_α
		Both simple	3·14	9·87
		Both encastré	6·28	39·5
		Simple and encastré	4·49	20·2
		Free and encastré	4·01	13·8
		Free and encastré	6·99	
		Both simple	4·25	13·3
		Both simple	5·4	
		Both encastré	5·9	

* See footnote *, p. 61.

Elevation and B.M. diagram	Plan	End fixity in twist*	m_β	m_α
		Both simple	5·5	18·34
		Both simple	7·65	22·8
		Free and encastré	6·43	
		Both simple	7 08	

* See footnote *, p. 61.

Paper No. 5959

"A Foundation Failure due to Clay Shrinkage caused by Poplar Trees"

by

Alec Westley Skempton, D.Sc.(Eng.), A.M.I.C.E.

(Ordered by the Council to be published with written discussion) †

SYNOPSIS

In 1942, serious cracking developed in the 65-foot-high wall of a theatre, built in 1928, situated in north London. By 1947, the main crack had attained a maximum width of nearly 2 inches. The foundations of the wall were 5 feet 6 inches wide, at a depth of 4 feet below ground level, and rested on firm clay with average liquid and plastic limits of 70 and 23 respectively. Investigations showed that the cause of the cracking was shrinkage resulting from drying of the clay by the roots of a row of Lombardy poplars, planted in 1930, at an average distance of 35 feet from the wall. This distance is equal approximately to the average height which the trees had attained in 1942. The wall was underpinned in 1947 and no subsequent movements have been observed.

No cracking has occurred in a neighbouring school (built in 1893), although one wall of this school is as close to the trees as the wall of the theatre. The school has a basement, however, and the footings of the school wall nearest to the trees are at a depth of about 9 feet. One wall in another part of the school is founded at a depth of not more than 4 feet, but these shallow footings are at least 62 feet from the trees. In Table 2 of the Paper these facts are compared with, and are shown to be closely similar to, the data previously published on buildings at Northolt where cracking was caused by the roots of black poplars.

The Paper concludes with some suggestions for the "safe distance" from trees of buildings on shallow foundations in clay soils, and with a recommendation that if houses and other small structures are to be built within this "safe distance" from trees, they should be founded on short-bored piles, unless a basement can be provided.

INTRODUCTION

THE danger of cracking in structures with shallow foundations on clays, owing to seasonal volume changes of the soil, is now becoming widely recognized, primarily as a result of the work carried out by the Building Research Station within recent years. Shrinkage and swelling movements, in the summer and winter respectively, have been shown by direct observation to extend to depths of the order of 4 or 5 feet in clays in southern and eastern England, where the vegetational cover is grass.[1] These seasonal

† Correspondence on this Paper may be accepted until the 15th and will be published in Part I of the Proceedings. Contributions should be limited to about 1,200 words.—SEC. I.C.E.

[1] The references are given on p. 83.

volume changes diminish with increasing depth, and experience has shown that the movements of footings 3 feet or more from the surface are so small as to cause no damage to houses or other buildings founded at this depth. In the immediate neighbourhood of a tree or group of trees, however, the seasonal movements are greater, and extend to a greater depth than in the case of a grass-covered clay soil. This difference results from the fact that, during the summer, the amount of water used by the tree in transpiration usually exceeds the amount of rainfall within the area of ground containing the tree roots. As a broad generalization, it may be said that the seasonal movement of the surface of a grass field, in the clays of south-east England, is about $1\frac{1}{2}$ to 2 inches and is the result of moisture changes to a depth of about 5 feet; but the seasonal movements of the ground surface near a tree in the same region may be as great as 3 or 4 inches, and this movement is the result of moisture changes typically extending to depths of 8 to 10 feet.[2] Similar moisture changes occur in sandy soils, but result in little volume change, and hence no foundation troubles arise in such soils from this cause.

The movements of clay soils are naturally greatest during a year with an exceptionally dry summer, whilst in an unusually wet summer they may be considerably less than the values mentioned above. Nevertheless, within the past 20 years there have been at least four summers when the lack of rainfall would cause movements of this order.[2]

These facts give rise to various questions, the answers to which are of considerable importance to the engineer and architect. Assuming, for example, that the foundations are the normal strip or spread footings at a depth of 3 feet on a clay soil, it is necessary to know how close to the building a tree of any particular species can safely be planted, or how close a building can be placed to existing trees. If, on the other hand, a new building is to be closer to a tree than this " safe distance " for shallow footings, it is necessary to know to what extra depth the foundations should be taken to avoid damage from the roots.

At least partial answers to these questions are now beginning to emerge, as will be seen in the section " Practical Recommendations " later in the Paper, but further advance depends primarily on the publication of a number of case records describing foundation failures due to shrinkage caused by tree roots, and it is for this reason that the present Paper has been written.

Site Conditions

In January 1947, the Author was consulted with regard to the serious cracking in the wall of a large theatre at Stamford Hill, in north-east London. The main crack in this wall can be seen in *Fig. 1* (facing p. 72).

The theatre was built in 1928 on an almost level site. The greater part of the building consists of a steel frame structure, with the stanchions on concrete footings founded at a depth of 16 feet below ground. Behind

the proscenium arch, however, the structure is built with brick load-bearing walls (see *Figs 2*). The back wall of the theatre is 65 feet high, carrying a reinforced-concrete roof and two floors spanning 30 feet over the stage. This wall has a thickness of 27 inches at its base, decreasing in steps to a thickness of 13½ inches at the top. The northern part of the wall is founded at a depth of 14 feet, owing to the presence of a basement beneath this side of the building, but the southern part of the wall is founded at a depth of 4 feet on a concrete strip footing 5 feet 6 inches wide. The average pressure exerted on the ground under this footing is 1·3 ton per square foot. Projecting from the back wall is a brick structure, containing a ventilation inlet duct, founded at a depth of 9 feet, and a small brick-walled shed containing a booster pump. The strip footings of the walls of this pump room were originally placed at a depth of about 2 feet 6 inches and the foundation pressure is rather less than ½ ton per square foot.

The adjacent site, to the south-east, is occupied by a school, built in 1893. The school has a basement running under the whole length of the side facing the theatre and extending under the building for a width of 34 feet, as shown in the plan, *Fig. 2* (*a*). The floor of the school basement is approximately 8 feet below ground level and the footings of the walls are presumably at least 1 foot below floor level. The area behind the theatre is completely paved with tarmac running right up to the wall dividing the two sites, and the ground around the school is also paved for the greater part, forming a playground for the children.

Borings at the positions shown in the plan revealed about 5 feet of firm brown clay, containing occasional stones, followed by brown London Clay. The deepest borehole extended 20 feet below ground level and showed brown London Clay to this depth. It is probable that the change from brown to blue London Clay occurs at a rather greater depth and, from well records in the neighbourhood, it is known that the total thickness of the London Clay is 80 to 100 feet. The upper layer of brown clay and stones is probably a solifluxion deposit consisting of drifted London Clay mixed with pebbles from the older Boyn Hill terrace gravels.[3]

A 6-inch-diameter drain, set in concrete, runs parallel to and at a distance of 12 feet from the wall. The trench for this drain was made 11 feet deep, and backfilled with clay. The trench was opened up, the drain laid, and the backfilling placed during the period of construction of the theatre.

At the time of the investigation, in February 1947, ground-water level was standing at a depth of 6 feet below the surface. It was reported to the Author that, in the summer months, ground-water level was at a depth of about 8 to 10 feet.

Case History

The theatre was built in 1928. The large unadorned back wall of the theatre is not attractive in appearance, and in 1930 a row of Lombardy

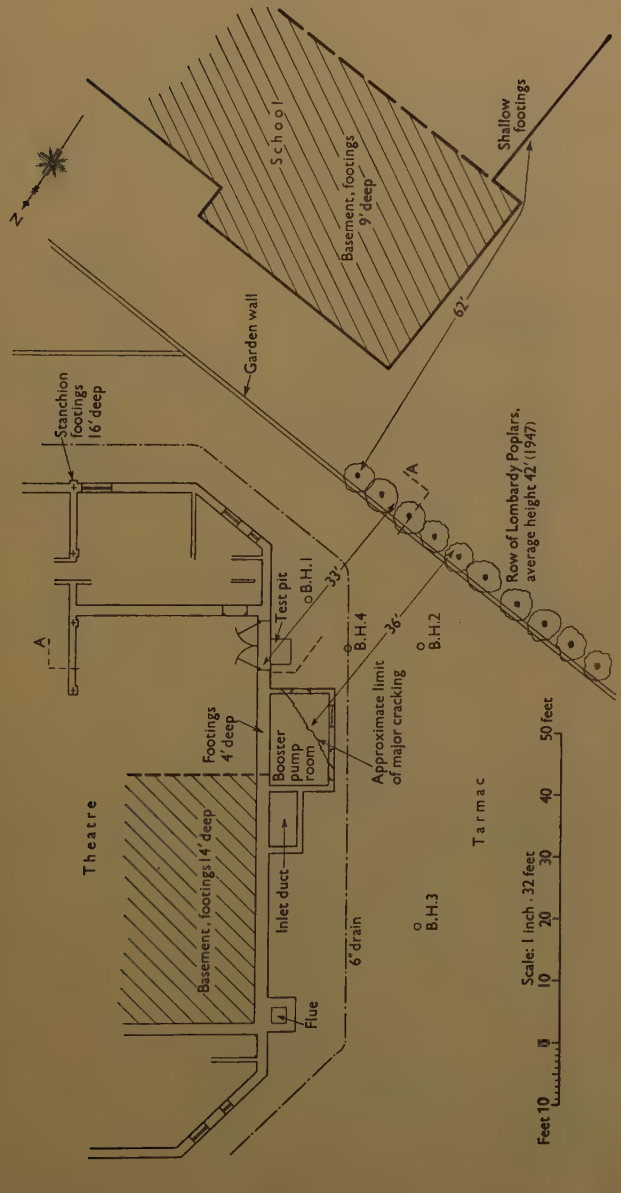
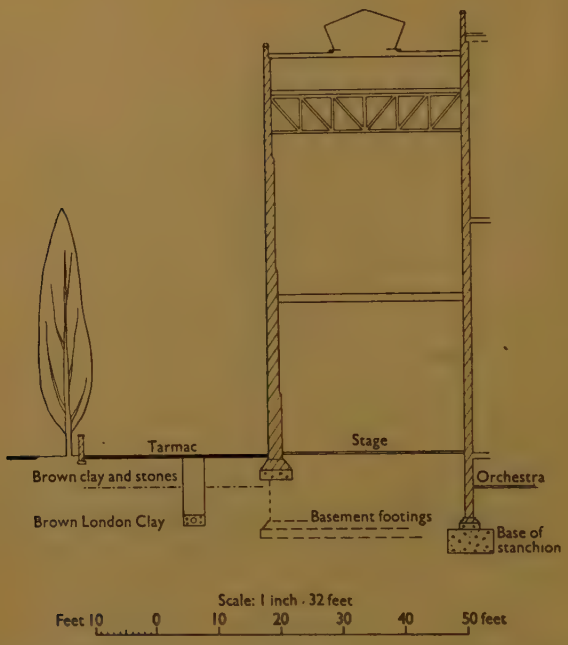


Fig. 2 (a)

SITE PLAN

Fig. 2 (b)



Cross-Section through Back of Theatre on line AA in *Fig. 2 (a)*

poplars was planted in the position shown in *Fig. 2 (a)*, evidently to act as a visual screen.

Some slight cracking is stated to have been seen in 1935 ; but in 1942, when the poplars were about 15 years old (assuming they were planted at the age of about 3 years), rather serious cracking was observed in the back wall of the theatre and in the walls of the booster pump room. In October 1942, the footings of the pump room were underpinned to a depth of 4 feet. After 1942, the crack in the back wall of the theatre continued to increase in magnitude and in the early months of 1947 the maximum width of the crack, near the roof, was about $1\frac{3}{4}$ inch. Daylight could be seen through the crack when standing inside the building. By this time steel channel sections had been bolted on the wall to act as ties across the crack. Subsidiary cracking was visible at several places on the south corner of the theatre near the doors leading to the stage, and on the roof above. The booster pump room had also continued to suffer, in spite of the underpinning.

In 1947, the poplars were about 20 years old, ranging in height from 30 to 55 feet and in girth from 18 to 27 inches. The average height of the trees was 42 feet.

A careful examination showed that the cracks in the back wall and the south corner of the theatre were caused by settlement of the southern part of the wall relative to the northern part. The greatest settlement was located at and near the south corner. The whole southern part of the wall was, in fact, rotating towards the south about a point approximately on the line of the south wall of the basement and a few feet above the ground. The outer walls of the pump room had settled and tilted outwards.

From the point of view of the general problem of the effect of tree roots on foundations in clay soils, it is of interest to note that, in contrast to the south-west end of the theatre, no cracks or distortions could be seen in the school building.

As previously mentioned, the Author's investigations were carried out in February 1947. The conclusion reached was that the settlements of the south corner and of the pump room were due primarily to the shrinkage of the clay, caused by the drying action of the poplar roots. The reasoning upon which this conclusion is based will be discussed in the next section of the Paper. To complete the case history, it is only necessary to add that, on account of the increasingly severe nature of the structural damage, the drastic remedial measure was recommended of underpinning the southern part of the wall to a depth equal to that of the northern part, namely, 14 feet. Owing to various circumstances, this work could not be carried out until November 1947. During the summer of 1947 the cracking was observed to widen still more, but during the 6-year period since the completion of the underpinning no further movement has been recorded.

Site Investigation

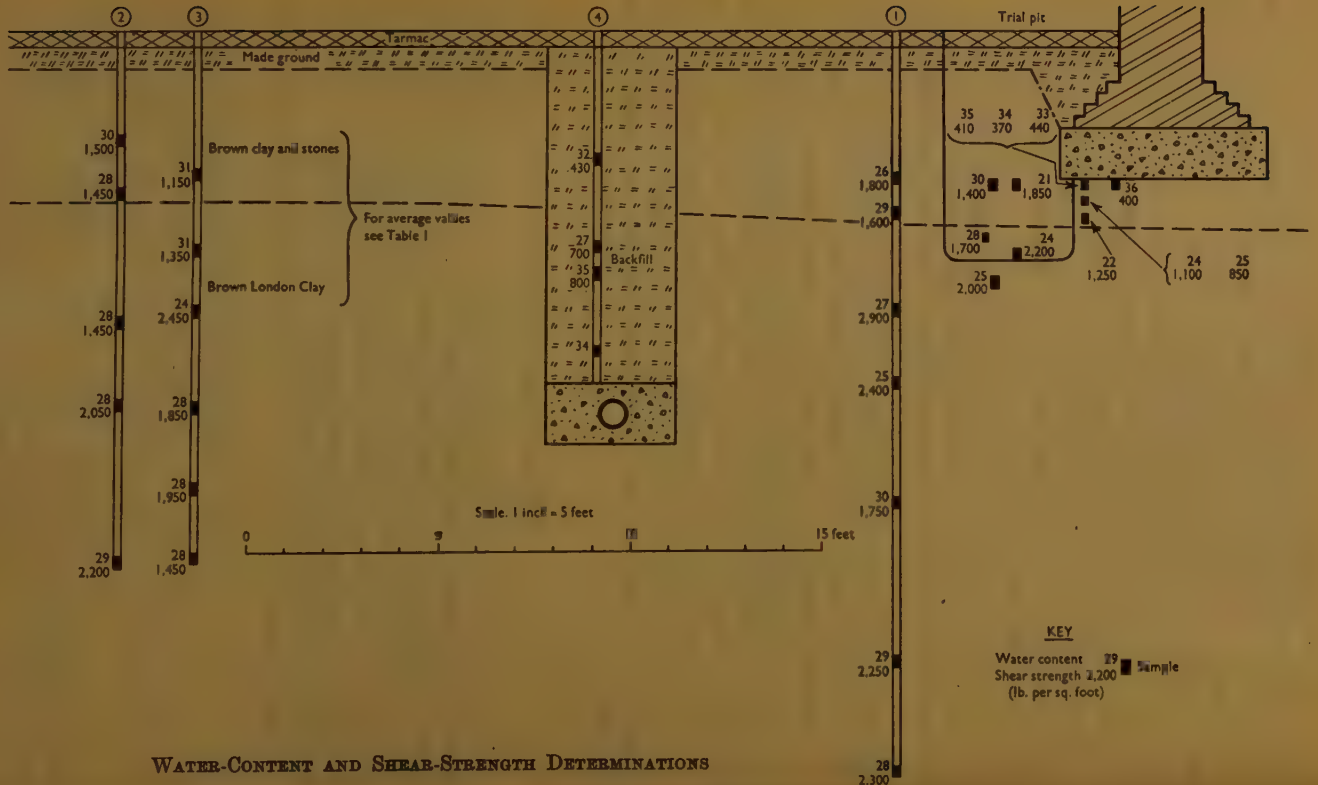
Four borings were made to determine the sequence of strata, and samples were taken at intervals of about 2 feet. A test pit was also put down to a depth of 6 feet immediately adjacent to the wall, in the position shown in *Fig. 2 (a)*, in order to investigate in detail the condition of the soil under the footings. The results are given in *Fig. 3* and are summarized in Table 1. Shear strength was taken as one-half the compression strength as measured in the undrained triaxial test.[4]

The footings of the back wall of the theatre and of the pump room were

Table 1

Material	Depth : feet	Water content	Liquid limit	Plastic limit	Shear strength : lb. per square foot
Brown clay and stones .	0–5	27	55	22	1,500
Brown London Clay .	5–20	28	85	24	2,000
Trench backfill . . .	0–9	32	—	—	650

Fig. 3



WATER-CONTENT AND SHEAR-STRENGTH DETERMINATIONS

Fig. 1

MAIN CRACK IN THEATRE WALL

Note poplars and school (partly hidden by trees). Photograph taken in November 1947, during the underpinning operations.

founded on the stratum of brown clay and stones, but at a depth of 14 inches below the footings, the brown London Clay is encountered. Where found in the boreholes and in the test pit, except in a thin layer immediately below the footings, the material described as brown clay and stones was firm, with an average shear strength of 1,500 lb. per square foot; the lowest strength of eight samples was 1,150 lb. per square foot. In contrast, the clay immediately below the footing was quite soft, with a strength of only 400 lb. per square foot. At a depth of 4 inches below the base of the footing the strength was 1,000 lb. per square foot, and 12 inches below the footing the strength was 1,250 lb. per square foot—a value within the range of strength found in the clay remote from the footing. It is therefore clear that, at the site of the test pit, the clay is very considerably softer in a thin layer immediately under the footing than elsewhere. It is particularly to be noted that two samples taken in the test pit at the level of the base of the footing and only 2 feet from its edge had a strength of about 1,600 lb. per square foot (almost equal to the average for the whole stratum).

The brown London Clay showed no significant variation in properties at any point from which samples were taken, except that the water contents in the upper 4 feet (that is, in the zone between 5 and 9 feet below ground level) were slightly less than those below 9 feet. The actual average values were 26·5 and 28·5 respectively.

In the test pit, living poplar roots were found, varying from hair thickness to about $\frac{3}{16}$-inch diameter. During the underpinning operations, many roots were seen in the sides of the pits. These were encountered to depths of 6 or 7 feet below ground level.

Analysis of Failure

After a consideration of four alternative explanations of the observed settlement of the southern part of the back wall of the theatre, and of the pump room walls, the Author concluded that the movements were due primarily to the drying, and consequent shrinkage, of the clay, by the action of the poplar roots. It would be tedious to enter into a lengthy discussion of the other three alternative explanations, but they will be indicated briefly in the following notes in order to show the lines of argument.

(1) *Normal seasonal shrinkage.*—Although this is a widespread phenomenon in clay soils, affecting houses [5] and roads,[6] it cannot possibly account for the settlements of the theatre wall or of the pump room after underpinning. Even had the ground adjacent to these walls been grass-covered, the seasonal movement below a depth of 4 feet (that is, below foundation level) are known to be very small.[1] But, in fact, the area adjacent to the walls is completely paved with tarmac, and this would greatly reduce the seasonal water-content changes (provided that no trees were present).

(2) *Settlement under foundation loads.*—No explanation of the settlements in terms of foundation loads can fully account for the following points:

(*a*) the severe movements in the pump-room walls, the foundations of which applied a pressure to the ground of less than ½ ton per square foot;

(*b*) cracking only became serious in 1942, 14 years after construction, but then continued to increase in magnitude;

(*c*) the width of the main crack in the theatre wall increased during the summer of 1947, whereas no appreciable movements had been noticed in the early months of that year.

With regard to (*a*) and (*b*), consideration has to be given to the thin layer of soft clay under the footing and to the main body of unsoftened clay. A possible explanation of the occurrence of the soft layer, as will be shown later, is that it was the result of softening in winter of clay which had shrunk away from the footing in summer. But it may be assumed as an alternative that the clay softened in the bottom of the foundation trench either before constructing the footings or by absorbing water from the freshly laid concrete. If the foundation load was sufficient to cause lateral plastic flow or consolidation settlement in this thin layer, the resulting movement would have been greatest at the south corner and almost zero at the centre-line of the building where the wall joins the part supported on the deep foundations. Hence it is conceivable that little pressure had been brought to bear on the clay near the site of the test pit, the wall arching over from the deep foundations to some point towards the south corner. In fact, however, the factor of safety against lateral plastic flow of the soft layer is 3½ (see Appendix I) taking the average foundation pressure of 1·3 ton per square foot; whilst if, in the extreme limit, about one-half of the southern part of the wall is virtually not pressing on the clay, the factor of safety against lateral flow in the clay layer beneath the southern quarter of the wall is still of the order 1·5. Moreover, if this were the cause of the settlements, they would have been completed within a few years after construction; and, finally, the load from the pump-room walls, being less than ½ ton per square foot, could not possibly cause any appreciable lateral flow, the factor of safety in that case being at least 5·0.

Similar arguments apply against settlements in the main body of the clay. The factor of safety with respect to a complete failure in shear of the underlying clay is 4¼ for the theatre wall (see Appendix II), and the corresponding final settlement of London Clay would be expected to be of the order 1⅓ per cent of the footing width, or 0·8 inch.[7] This is the absolute settlement of the southern part of the wall. The northern part of the wall will have settled by a somewhat smaller amount and it is unlikely that the differential settlement could appreciably exceed ½ inch.

This movement would not be sufficient to cause any appreciable cracking [8] and, moreover, these settlements would be largely completed within a period of 5 years after construction,* whereas the cracking was not serious until 14 years after construction and became worse in the succeeding 5 years. In addition, of course, the settlements of the pump-room walls, due to loading the clay, would be still smaller, and they would be completed even sooner than in the case of the theatre wall.

The above calculations, taken together with the observed increase in crack width in the *summer* of 1947, suggest strongly that the trouble cannot wholly be attributed to movements consequent upon the foundation loads.

(3) *Movement toward the drain trench.*—The drain trench extends to a depth of 9 feet from ground level to the top of the concrete block in which the 6-inch-diameter drain is set. The trench is 10 feet from the edge of the theatre-wall footing and runs within a foot of the pump-room foundations. The northern part of the theatre wall is founded at a level below the trench and could not therefore be influenced. But the southern part of the theatre wall, and the pump-room walls, are founded 5 feet above the concrete filling at the bottom of the trench. It may, then, be suggested that the settlements were caused by movements of the clay towards the trench, and the fact that no serious cracking appeared for 14 years after construction could be attributed to the well-known progressive softening that takes place in fissured clays, such as the brown London Clay. When such clays are allowed to expand laterally (as was possible in the present case during excavation of the trench) some of the fissures open slightly and, consequent upon the entry of water, an internal softening process is initiated. Experience has shown that after a period of about 20 years the brown London Clay can soften to an average strength of approximately 400 lb. per square foot.[9]

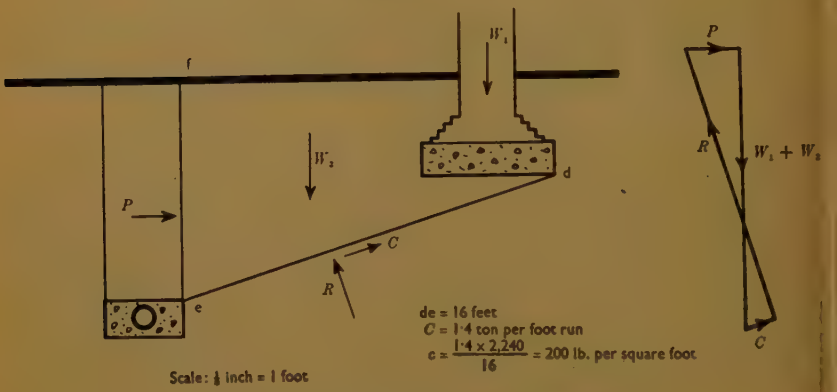
This softening occurs, however, only where the fissures are free to open as a result of lateral expansion towards the excavation. It could not occur in clay beneath a foundation. The length of a slip plane running from the inner edge of the theatre-wall footing down to the bottom of the trench is 16 feet (see *Figs 4*). Of this, a length of at least 6 feet would be subjected to compression stresses from the foundation and therefore would not be able to soften by infiltration of water along fissures. The average shear strength on the slip plane is thus not less than about 800 lb. per square foot, and a conservative estimate shows that the strength required to prevent movement along the slip plane is 200 lb. per square foot (see Appendix III). The factor of safety against such movement is therefore at least 4, and more probably it is appreciably greater. So far as the pump-room foundations are concerned, the factor of safety against

* Observations on buildings with footings up to about 10 feet wide, on London Clay, have shown that the rate of settlement becomes almost zero at about 5 years after construction.

a shear failure in the clay is at least $2\frac{1}{2}$, even if the clay beneath the footing has a strength of 400 lb. per square foot and, as shown above, the actual strength is certainly greater.

Thus it must be concluded that the settlements cannot be attributed

Figs 4



ANALYSIS OF SLIP TOWARDS DRAIN TRENCH

to this cause; and this is to some extent confirmed by the observation that the cracking increased during the summer of 1947. Had the movements been the result of shear in the softened London Clay they would be greater in the winter rather than in the summer.

Shrinkage of Clay caused by the Tree Roots

It has been seen that none of the three foregoing alternative explanations of the cracking are free from objection. In contrast, an explanation in terms of shrinkage of the clay caused by the drying action of the poplar roots is in accord with all the observations. When considering normal seasonal shrinkage two facts were encountered that could not be accounted for, namely the 4-foot depth of footings and the covering of the area adjacent to the walls with tarmac. But there is no reason why shrinkage caused by the tree roots should not proceed in the clay below 4 feet, since the roots were actually seen at depths down to 6 or 7 feet. Moreover, by restricting the infiltration of rain-water, the tarmac would cause the roots to take more water out of the clay than would be the case without an impermeable covering of the ground.

Two of the objections to an explanation in terms of settlements caused by the foundation loads were, firstly, that the loads appeared to be too small to account for the movements, especially in the pump room, and secondly that no serious cracking occurred until 14 years after construction. In terms of shrinkage by the tree roots, however, the foundation

loads are irrelevant ; shrinkage takes place even with no load on the clay. The delay of 14 years before any major cracking occurred is readily explained by the increasing outward spread of the roots as the poplar trees were growing. The distance from the trees to the pump room and to the centre of the southern part of the theatre wall is about 35 feet. But the trees, when planted, were only saplings, and it would obviously be many years before their roots would extend as far as 35 feet from the row. In 1942, when the cracking first became serious, the trees were about 15 years old and 30 to 35 feet high, on the average.* By this time it is almost certain that the roots would have reached the clay beneath the footings, since it is known that, in general, roots extend to a distance of rather more than the height of the tree.[11, 12] The increase in cracking during the years 1942 to 1947 would, on this basis, be explained simply by a further increase in the number of roots in the clay beneath the footings and to the increased water requirements of the actively growing trees. Moreover, the summers of 1942 to 1945 were drier than average, and so was the summer of 1947.

The fact that the crack width increased during the summer of 1947 is thus readily accountable for ; although it presents a difficulty in explaining the movements in terms of foundation loads and also in terms of movements resulting from softening of the London Clay and consequent slipping towards the drain trench.

The slightly lower average water content in the top 4 feet of the London Clay (26·5), as compared with that at greater depth (28·5), may be a direct confirmation of the drying action of the roots. But it will be recalled that the site investigation was carried out in February 1947, when the trees were not in leaf, and any drying effect would then be at the minimum.

With regard to the thin layer of soft clay immediately beneath the footing of the theatre wall, as found in the test pit, it is not at once obvious in what manner this can be correlated with shrinkage caused by the tree roots. But shrinkage is not uniformly distributed along the length of a footing, nor across its width, and in two cases investigated by the Building Research Station, during the late summer months, a clear gap of the order of $\frac{1}{4}$-inch width has been found in places between the underside of footings and the clay, where intense shrinkage has occurred as a result of tree roots.† The structural conditions of the theatre wall are particularly well suited to the formation of such gaps near the pump room, since the wall is prevented from settling and following up the shrinkage movement near its junction with the part of the wall founded at a depth of 14 feet. Near the south corner, however, the wall can follow these movements, once the crack has formed in the brickwork. It would consequently be

* The measured average height of 42 feet at an age of 20 years agrees well with the data for poplars given by Brenda Colvin.[10] This data has therefore been used to estimate the average height of the trees at an age of 15 years.

† Personal communication from Dr L. F. Cooling.

expected that few, if any, gaps could develop near this corner but, on the contrary, it is not difficult to imagine these gaps occurring in the summer beneath the footings of that part of the wall to the north of the test pit. The surface of the clay in such gaps would be free to absorb water when, in the autumn, the trees cease to transpire and more water becomes available from the increased rainfall typical of this time of the year. The presence of the soft clay may therefore be attributed tentatively to shrinkage followed by absorption of water by the clay in the autumn and winter.

If this account of the thin layer of soft clay be accepted then it follows that the partial removal of support by shrinkage would cause an increase in load on the clay in the more southerly end of the wall where the footings were able to follow the downward movement of the clay in the summer. This increase in load, consequent upon a redistribution of pressure along the length of the footing, would result in a somewhat larger settlement than that calculated previously; and settlement under load has almost certainly played a subsidiary role in the cracking of the theatre wall. The conclusion remains, nevertheless, that shrinkage due to the drying action of the roots was the primary cause of the cracking in this wall; and in the case of the pump-room walls, shrinkage would seem to be the only cause which is in accordance with the observations and calculations.

The absence of any cracking in the school building might be adduced as a fact which is not concordant with an explanation in terms of shrinkage caused by the roots, especially since the north wall of the school is only 18 feet from the row of poplars. But the northern side of the school is founded at a depth of rather more than 9 feet, and this is not only deeper than the roots were found in the clay, but is certainly below winter ground-water level and, in all probability, deeper even than summer water level. And it is, of course, quite obvious that no shrinkage of the clay can take place below ground-water level.

Reference to the plan in *Fig. 2 (a)* will show, however, that part of the school on the side remote from the theatre is not founded on a basement. Therefore the above argument cannot hold good for this part of the school building. But the shortest distance which could be followed by roots from the trees to the school walls on shallow footings is about 62 feet, and the absence of cracking here may be simply because the roots had not reached so far.

The northern part of the theatre wall, founded at 14 feet, the ventilation inlet duct, founded at 9 feet, and the stanchion bases, founded at 16 feet, would all be immune from shrinkage settlement since their footings are below ground-water level.

Conclusions and Summary

After considering four alternative explanations of the cracking of the back wall of the theatre and of the pump-room walls, it is concluded that

the drying and shrinkage of the clay beneath the footings of these walls is the only explanation in accordance with all the observed facts of the case.

The footings of the theatre wall and of the pump room (after underpinning) were founded at a depth of 4 feet. Poplar roots were discovered in the clay to a depth of 6 or 7 feet; that is to say, to a depth of 2 or 3 feet below the footings. The average water content and Atterberg Limits of the clay within this zone are:

water content	27
liquid limit	70
plastic limit	23.

In 1942, when cracking in the walls first became serious, the poplars, which had been planted in 1930, had an average height of 30 to 35 feet. The distance from the trees to the position of worst cracking in the pump room (see *Figs 2*) is 36 feet. The southern part of the theatre wall lies between 20 and 45 feet from the trees, or at an average distance of 33 feet. Since the height of the trees in 1942 has to be estimated, and since the distance to the theatre wall ranges between rather wide limits, it is justifiable to quote these figures only to the nearest 5 feet. In that case, the average tree height and the average distance from the trees to the cracking (theatre wall and pump room) are both 35 feet.

In 1947, when the poplars had an average measured height of 42 feet, no cracks could be observed in the school building, on the south side of the trees. That part of the school nearest the trees is founded on a basement with footings a little more than 9 feet deep. These footings are below the depth at which the roots were found in the clay, and are also below winter ground-water level. They are probably also below summer ground-water level. The part of the school on shallow footings is everywhere more than about 62 feet from the poplars—a distance equal to $1\frac{1}{2}$ times the average tree height.

From these observations, three conclusions may be drawn. First, that the shrinkage of a clay soil caused by the roots of a row of poplar trees is sufficient to cause serious cracking in walls on shallow foundation at a distance from the trees equal to their height; but, secondly, that no damage will occur in walls on shallow foundations at a distance from the trees equal at the most to $1\frac{1}{2}$ times their height. It is probable that had the school walls on shallow foundations been closer than 62 feet from the trees they would not have cracked. The significance of the ratio $1\frac{1}{2}$ is that walls at this distance from the trees were not damaged; and in an empirical study of this nature it is almost as important to obtain data on the absence of cracking as on the incidence of cracking. The third conclusion is that, in relation to shallow-rooting trees such as poplars, a basement is probably a safeguard in most cases, whilst no trouble from tree

TABLE 2.—SUMMARY OF DATA

Site and reference	Clay and ground cover	Index properties of clay (average values)			Date of cracking	Depth of footings	Depth to summer G.W.L.	Trees		Average dist. to major cracking (L_c): feet	$\frac{L_c}{H}$	Average dist. to uncracked walls † (L_0): feet	$\frac{L_0}{H}$
		Water content	Liquid limit	Plastic limit				Distribution and species	Average height (H): feet				
Northolt (Ward, 1947)	London Clay with grass cover	29	84	25	1945	3′ 8″	Not known	Double row of black poplars	32	34	1·1	38	1·2
Stamford Hill (present Paper)	Drift clay over London Clay, with tarmac cover	27	70	23	1942	4′ 0″	10 ft. (approx.)	Single row of Lombardy poplars	35	35 (approx.)	1·0 (approx.)	—	—
					1947	*			42	—	—	62	1·5

* Depth not known, but probably not more than 4 feet.

† The dimensions given in this column are not critical: if there had been walls somewhat closer to the trees than L_0 they might not have shown cracks.

roots can possibly be expected with foundations which bear on clay below summer ground-water level.

The first two conclusions are in agreement with the only other published record of damage caused by a row of poplars.[11] At this site, at Northolt in north-west London, a row of black poplars was planted in 1938. In the summer of 1945, when the trees were 10 years old and had reached an average height of 32 feet, severe cracking developed in an adjacent two-storey brick building. The footings of this building were 3 feet 8 inches deep, founded on brown London Clay, and the date of construction was 1923. Major cracking extended to a distance of 34 feet from the trees, that is, a distance equal to 1·1 times the tree height. Three other buildings on the site were placed at an average distance of 38 feet from trees of the same species which had also been planted in 1938, and these buildings were not cracked. This distance equals 1·2 times the tree height. In the building mentioned above, minor cracking was observed to a maximum distance of 43 feet from the trees, but it is probable that this was consequent upon the major movements of the walls nearer the trees and not due directly to shrinkage. As a summary, the relevant data from these two sites are collected together in Table 2.

Practical Recommendations

Finally, it is of interest to give some consideration to a provisional rule recently suggested by the Building Research Station,[1] namely, that a " safe distance " equal to the height of the trees should be left between the trees and a building with shallow foundations on clay soil. This rule is clearly supported by the data in Table 2. But it must be noted that this data applies (*a*) to a row of trees ; and (*b*) to trees with high water requirements : a category including poplars, elms, limes, and oaks. Now, as Ward [11] has pointed out, the roots of trees planted in a row may be expected to extend to greater distances than the roots of a single tree of the same height and species. Thus, where a single tree is concerned, even if it belongs to the category of species with a high water-requirement, the " safe distance " may be rather less than the tree height.

In addition to the difference between a single tree and a row, consideration should be given to the possibility that the " safe distance " may be a still smaller proportion of the tree height for those trees with comparatively low water-requirements. It is at present impossible to put forward any definite rules for this category ; but the Author would suggest that ash, birch, hornbeam, and all evergreens might be included, and he would be surprised if, for single trees of these species, the " safe distance " were found to be more than two-thirds of the height.

The foregoing remarks are, however, still a matter of opinion, and reliable solutions to the problem will be obtained only when a number of

accurate case records have been published. Nevertheless, from the information available,[12] it is already clear that if a building is to be placed within a distance from trees (especially poplars, elms, limes, and oaks) equal to or less than the height to which the trees may be expected to grow, and if the foundations are shallow and are on clay, then there is a distinct possibility that trouble may be experienced, owing to shrinkage.

The two practical alternatives available to the designer, in order to avoid such trouble, are (1) that the trees closer to the building than the " safe distance " should be cut down ; or (2) that the building should be provided with foundations sufficiently deep to obtain a bearing on ground which is not appreciably influenced by the tree roots. In many cases it is most undesirable to cut down the trees, and the designer has then to decide by what means the foundations can be deepened without adding excessively to the cost.

The value of a deep foundation has been clearly demonstrated in the present investigation, since the school suffered no damage (although as close to the poplars as the theatre) owing to the fact that it had a basement, whereas the theatre wall was on footings only 4 feet deep. Where a basement can be usefully included in the design it should act as a reasonably good safeguard. Few modern buildings of small height, however, are provided with a basement ; but the beneficial effects of a deep foundation can be obtained by the use of piles. In particular, for houses and other two- or three-storey buildings, the " short-bored pile " foundation seems to be satisfactory. Several publications from the Building Research Station have described this foundation.[13, 14, 15, 16] Typically, the piles are about 12 feet long, 14 inches in diameter, and will carry a safe load of 6 to 8 tons. For a number of houses and schools recently constructed it appears that the short-bored-pile foundation is not more costly than the traditional strip-footings 3 feet deep. Yet, by obtaining their bearing at a greater depth, the bored piles are markedly superior from the point of view of guarding against clay shrinkage caused by tree roots. Indeed, if the piles penetrate to a depth of several feet below summer ground-water level, they will derive much of their bearing capacity from clay which cannot be subject to shrinkage. This type of foundation has not yet been fully tested in the light of long-term practical experience, but logically it would seem to be a sound expedient and will almost certainly find increasing application in areas with clay soils.

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The Paper is accompanied by one photograph and five sheets of diagrams, from which the half-tone page plate and the Figures in the text have been prepared, and by the following Appendices.

APPENDIX I

Plastic Flow of Soft Clay Layer

The two-dimensional problem of plastic flow in a clay layer between two surfaces has been solved by Prandtl (in 1923) and the solution is readily accessible in Nadai's text book on " Plasticity."[17] Referring to *Fig. 5*, a strip foundation of width $B = 2b$ is placed at a depth D below the surface, and it is required to find the average foundation pressure causing plastic flow in a layer of clay of thickness $H = 2a$ and with a shear strength c.

With the origin of co-ordinates at the centre of the layer and on the centre-line of the foundation, the normal stresses at any point within the clay layer, when plastic flow is taking place, are :

$$\sigma_x = K - c\frac{x}{a} - 2c\sqrt{1 - \frac{y^2}{a^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\sigma_y = K - c\frac{x}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where K denotes a constant of integration to be determined by the boundary conditions.

On the plane lm beneath the edge of the footing, the horizontal active thrust developed by plastic flow in the clay layer is :

$$P_1 = 2\int_0^a \sigma_x \, . \, dy$$

and, from equation (1) :

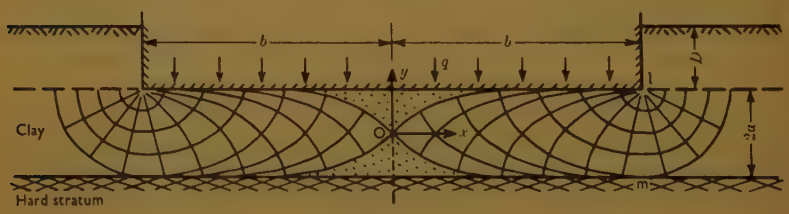
$$P_1 = K \, . \, 2a - c \, . \, 2b - c \, . \, a\pi$$

This thrust is in equilibrium with the passive pressure developed in the clay outside the plane lm. This passive pressure P_2 can be written in the form

$$P_2 = (K_p \, . \, c + \gamma D)2a$$

where K_p denotes the coefficient of passive earth pressure in the clay layer and γD the

Fig. 5



Approximate Slip Lines in a Clay Layer Beneath a Strip Foundation.
(Angle of shearing resistance in clay layer, $\phi = 0$).

overburden pressure at foundation level. Assuming, as a first approximation, that the slip lines in the passive zone are circular arcs, as shown in *Fig. 5*, then :

$$K_p = \pi$$

and hence $$P_2 = (\pi c + \gamma D)2a$$

From the equation of equilibrium $P_1 = P_2$ it then follows that :

$$K = c\left(\frac{3\pi}{2} + \frac{b}{a}\right) + \gamma D$$

Now the average foundation pressure causing plastic flow is :

$$q_f = \frac{1}{b}\int_0^b \sigma_y \, . \, dx$$

and from equation (2); $q_f = K - c\,\frac{b}{2a}$

Substituting the expression for K in the above equation, the required solution is obtained :

$$q_f = c\left(\frac{3\pi}{2} + \frac{b}{2a}\right) + \gamma D$$

If the actual foundation pressure is q, where q is less than q_f, then the corresponding factor of safety F is given by the equation :

$$q = \frac{c}{F}\left(\frac{3\pi}{2} + \frac{b}{2a}\right) + \gamma D$$

or

$$q - \gamma D = \frac{c}{F} \cdot N_c$$

This solution is approximate. A more rigorous treatment would result in an expression showing an increase in the bearing capacity factor N_c with increasing values of D. But for most purposes, and especially when, as in the present problem, $\frac{b}{2a}$ is large compared with $\frac{3\pi}{2}$, the foregoing solution is adequate. Since the critical slip surface in a deep bed of clay extends only to a depth of about $\frac{2B}{3}$ beneath the footing, it is clear that the thin-layer equations are restricted to cases where $\frac{b}{2a}$ is less than about 0·7. For thicker layers the normal bearing-capacity theories apply (see Appendix II).

In the case of the theatre wall $q = 1{\cdot}3$ ton per square foot (= 2,900 lb. per square foot) ; $\gamma = 125$ lb. per cubic foot ; $D = 4$ feet ; $2b = 66$ inches ; $2a = 2$ inches ; and $c = 400$ lb. per square foot. With these values, the factor of safety is found to be $3\frac{1}{2}$.

APPENDIX II

Bearing Capacity

If q denotes the foundation pressure beneath a strip footing of width B at a depth D in a clay with an average shear strength $\bar{c}$ and density γ, then the factor of safety F against ultimate failure is given by the expression [7] :

$$q - \gamma D = \left(5 + \frac{B}{D}\right)\frac{\bar{c}}{F}$$

provided (i) that D is less than $2{\cdot}5B$, and (ii) that the clay extends to a depth of at least $\frac{2B}{3}$ beneath the footing. The value of $\bar{c}$ may be taken as the average shear strength within a depth of $\frac{2B}{3}$ below the footing.

For the theatre wall, $D = 48$ inches and $B = 66$ inches. The density of the clay is 125 lb. per cubic foot and γD therefore equals 500 lb. per square foot.

The shear strengths of the clay in the different layers within a depth of $\frac{2}{3}B = 44$ inches below the footing are as follows (see *Fig. 3*) :

depth	c
0–2 inches	400 lb. per square foot
2–6 „	1,000 „ „
6–14 „	1,500 „ „
14–44 „	2,000 „ „

and hence the value of $\bar{c}$ is 1,750 lb. per square foot. The actual foundation pressure q is 1·3 ton per square foot, or 2,900 lb. per square foot, and hence, from the above expression, the factor of safety F is 4·2.

In the same way, it can be shown that the factor of safety of the pump-room foundation is about 2½, even if the average shear strength of the clay beneath the footings is taken to be as low as 400 lb. per square foot. Here the net load ($q - \gamma D$) is about ½ ton per square foot, the footings are 23 inches wide and, after underpinning, the depth D becomes 48 inches.

APPENDIX III

Slip towards Drain Trench

In London Clay which has been able to expand laterally towards an excavation, some of the fissures open up and an internal softening process is initiated. Field observations have shown that after a period of about 20 years (that is, from the date of construction, 1928, to the time of the investigation in 1947) the average shear strength of such clay can fall from its initial value of the order 2,000 lb. per square foot to a value of 400 lb. per square foot.[9] But in clay subjected to compressive stresses, as under a foundation, this softening process cannot occur. To obtain an estimate of the average strength along a potential slip plane de (see *Figs 4*), it can be assumed that of the total length of de (= 16 feet) at least 6 feet will pass through unsoftened clay beneath the footing. The strength along the slip plane is then made up as follows :

4 feet of brown clay and stones	c = 1,100 lb. per square foot
2 feet of unsoftened London Clay	2,000 „ „
10 feet of softened London Clay	400 „ „

and the corresponding average shear strength is 800 lb. per square foot.

The clay backfilled into the drain trench has a shear strength of about 650 lb. per square foot and a density of about 120 lb. per cubic foot. If the wedge def (in *Figs 4*) starts sliding down the plane de, a pressure will be developed in the backfilling which, in the limit, will attain the full passive pressure of clay with the properties given above. This passive pressure will have a value at least equal to that given by the expression :

$$P = \tfrac{1}{2}\gamma H^2 + 2cH$$

where γ denotes density, c the shear strength of the clay, and H the depth of the trench. According to this equation, $P = 7{\cdot}4$ tons per foot run. But the movement required to mobilize this full passive pressure may be rather large. Consequently, in order to obtain a very conservative estimate of the problem, it may be assumed that the only resistance offered by the backfilling is the gravity term in the foregoing equation ; or, in other words, the movement is assumed to be so small as not to have mobilized any appreciable shear strength in the backfilling. The pressure P is then 2·2 tons per foot run.

The weight of the theatre wall (W_1 in *Figs 4*) is 7·5 tons per foot run and the weight of the wedge def is 4 tons per foot run (W_2). If $P = 2{\cdot}2$ tons per foot run, then a simple polygon of forces, as shown in *Figs 4*, indicates that the average shear strength along de required for equilibrium is 200 lb. per square foot. But the average strength along the plane is about 800 lb. per square foot. The factor of safety against a slip of this type is therefore approximately 4.

CORRESPONDENCE

on a Paper published in Proceedings, Part I, March 1953

Paper No. 5877

"Numerical Solution of Some Problems in the Consolidation of Clay" †

by

Robert Edward Gibson, B.Sc.(Eng.), and Peter Lumb, M.Sc.(Eng.), Studs I.C.E.

Correspondence

Professor A. D. Ross commented that the method of computation presented in the Paper was both powerful and flexible. He believed, however, that the numerical process could be rendered even more attractive if certain special conditions were observed. The equation for one-dimensional consolidation when quoted in finite difference form was :

$$u_1 \simeq \beta (u_2 + u_4 - 2u_0) + u_0$$

which was equation (5) of the Paper. Now if, quite arbitrarily and as a special case, β was made equal to $\frac{1}{2}$ then the equation became :

$$u_1 \simeq \tfrac{1}{2}(u_2 + u_4)$$

Since β was $\frac{c_v \delta t}{\delta z^2}$ it was a simple matter to ensure that that factor was $\frac{1}{2}$ by choosing appropriate values of δt and δz. On that basis, the labour of computation was greatly reduced since the value of u at any station was always the mean of the previous values at adjacent stations. Table 1 showed how the case of *Fig. 2* could be calculated in that way in less than 3 minutes. It would be noted with that system,[13] only alternate values were significant and therefore only those were entered in Table 1, thus speeding the process. To compute $\overline{U}$, the missing values could readily be interpolated although even that was scarcely necessary when m was large. Table 1, of course, gave the same solution as *Fig. 2* with the difference that the intervals of time were doubled since β had been doubled.

† Proc. Instn Civ. Engrs, Part I, vol. 2, p. 182 (Mar. 1953).

[13] A. D. Ross and J. W. Bray, "The Prediction of Temperatures in Mass Concrete by Numerical Computation." Mag. Concr. Res. No. 1 (Jan. 1949), p. 9.

TABLE 1

Time intervals	Distance from drained boundary				
	0	δz	$2\delta z$	$3\delta z$	$4\delta z$
0	(100)	100	(100)	100	(100)
1	0	—	100	—	100
2	—	50	—	100	—
3	0	—	75	—	75
4	—	37·5	—	75	—
5	0	—	56·3	—	56·3
6	—	28·1	—	56·3	—
7	0	—	42·2	—	42·2
8	—	21·1	—	42·2	—
9	0	—	31·6	—	31·6
10	—	15·8	—	31·6	—

The only restriction was that for a constant value of c_v, there should be a unique numerical ratio between δt and δz, but that was no disadvantage ; if more values at shorter times were required it was only necessary to increase m, that is, to reduce δz, and there was little hardship in that since the process was so simple.

The use of a particular value of β became even more valuable in two- and three-dimensional cases. If, in a two-dimensional field with Cartesian co-ordinates, a station q in a square network was surrounded by four adjacent stations m, n, p, and r, and if β was arbitrarily made equal to $\frac{1}{4}$, then, by a similar argument, it could be shown that :

$$u_q \simeq \tfrac{1}{4}\{u_m + u_n + u_p + u_r\}$$

Likewise in the three-dimensional case, if $\frac{c\delta t}{\delta z^2}$ was made equal to $\frac{1}{6}$ then :

$$u_q \simeq \tfrac{1}{6}\{u_m + u_n + u_p + u_r + u_v + u_w\}$$

It would be noted that successive applications of that equation would give the complete solution in one operation without the use of the product solution indicated in equation (11).

Thus, in all cases, the value at a station at a time $t_1 + \delta t$ was the mean of the values at the surrounding two, four, or six stations at time t_1. That reduced the arithmetical working to the minimum and Professor Ross remarked that he had used those methods both in two and three dimensions for thermal problems with great convenience. He had also checked the accuracy in the limited number of cases for which exact solutions were available and the results had been gratifying.

There was the further point that that simplification enabled graphical methods to be used if so desired. In the one-dimensional case, a graphical solution was both obvious and simple, since the averaging process could be performed by drawing straight lines between alternate ordinates represent-

ing u at abscissae representing the distance z. Professor Ross had recently devised a simple mechanical procedure, based on a graphical construction, whereby with the aid of a drawing board, flag-pins, and elastic threads he could obtain solutions to two-dimensional problems with any boundary shape and with boundary conditions varying with time. Those solutions were obtained rapidly and without fatigue ; the accuracy was, of course, somewhat reduced but was still acceptable for most engineering problems. Most important was the fact that the operator could follow visually, in vector quantities, the progress of the operation and the manner in which the excess pressure in the pore-water (or the temperature in thermal problems) varied at each point in the field.

Mr R. T. Severn commented that the Authors had introduced a value $\beta = \frac{c_v \partial t}{\partial z^2}$ and suggested that the accuracy of the solution would depend on the value given to β. That was to some extent true, but J. Von Neumann had shown that, for a stable solution, β could not take any value, but should satisfy the inequality $\beta \leqslant \frac{1}{2}$. The value $\beta = \frac{1}{2}$ separated the stable region where errors decayed from the unstable region where errors grew. As an example, Mr Severn proposed that β was made equal to unity in equation (5) on p. 185, then :

$$u_1 \simeq (u_2 + u_4 - 2u_0) + u_0$$

The finite-difference scheme showed obvious instability (*Fig. 10*). Definite but less marked instability was shown in *Fig. 11* where $\beta = 0{\cdot}6$.

The Authors had appreciated the difficulty in obtaining pore-water pressure accurately, after long periods of time, by the forward integration procedure which they had used, and had referred to relaxation methods as resolving the difficulty. The main advantage of the latter method was that no restriction was placed on the value of β and therefore a large number of intervals in the time direction could be commensurate with any number of intervals in the space direction. As an example, the relaxation method had been used (*Fig. 12*) to compute values of u for the problem given by the Authors in *Fig. 2*. As expected, the error in the forward integration method became serious only at the end of the time range, and would continue to increase with time.

Mr R. F. Scott, of Cambridge, Massachusetts, observed that the Paper was the first, to his knowledge, to be published on the subject in English. He had been interested in such solutions for some time and would like to make some comments on the Authors' methods and results.

A considerable amount of work had already been done on the use of numerical procedures to solve diffusion equations, largely in the field of heat transfer, and it seemed to Mr Scott that it would be of value to an engineer interested in those methods to study some of the articles and books already published.[14]

[14] References 14 to 28 are given on p. 97.

Fig. 10

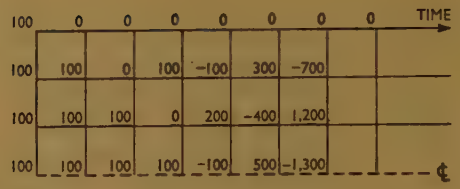


Fig. 11

Fig. 12

The Authors had developed equation (5) for the one-dimensional case, and assigned the symbol β to the factor $\frac{c_v \delta t}{\delta z^2}$. They had not mentioned that a certain limitation was imposed on the value of β, namely, that it might not exceed $\frac{1}{2}$ without the results oscillating and diverging. If a value of $\frac{1}{2}$ was used, which was an obvious first substitution, the simple equation :

$$u_0(t + \delta t) = \tfrac{1}{2}(u_2 + u_4) \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

was obtained, but on application of that expression the computer would find that the results would oscillate. That, and the convergence of the solutions, had been investigated mathematically by O'Brien, Hyman, and Kaplan [15] ; Leutert [16] ; and Hildebrand.[17] O'Brien, Hyman, and Kaplan [15] had given curves showing the desirability of maintaining the value of $\beta < \frac{1}{2}$.

The Authors had used equation (4) (p. 185) in order to obtain the average values of degree of consolidation given in their examples from the numerical values. Had they utilized a planimeter to determine the

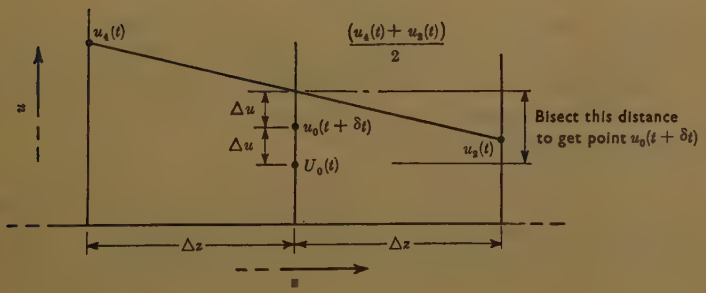
integrals or had a numerical integration approach been used ? The reason for the inquiry was that naturally the error of the method of integration affected the accuracy of the solution and the relative error of a planimeter varied inversely as the area.

On p. 187, the statement was made that "it is difficult to determine the pore-water pressure-distribution accurately after long periods of time." Mr Scott agreed with that, naturally, but would like to point out that if, for instance, the effect of differing rates of loading on the consolidation of the ground in any particular case was being studied, then calculations would have to be made only for the simple instantaneous loading case, and that in all other cases (except where excavation or unloading, with consequent swelling, was concerned) the desired values or curve could be obtained by superposition.

Some reduction of mental labour could be achieved for the one-dimensional case in two ways:—

(1) A graphical solution to the problem was available for $\beta = \frac{1}{2}$ or in a more complicated (and more accurate) fashion for $\beta = \frac{1}{4}$. For $\beta = \frac{1}{2}$ the graphical method was a simple averaging technique, and the procedure for $\beta = \frac{1}{4}$ was presented in *Fig. 13*. The proof was elementary. Such graphical methods gave the pore-water pressure curves with time directly.

Fig. 13

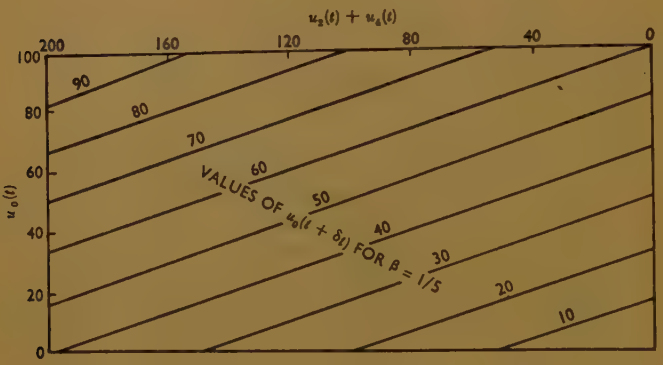


(2) A chart could be made up for the particular value chosen for β, giving a set of curves of different values of $u_0(t + \delta t)$ plotted against $u_0(t)$ and $[u_2(t) + u_4(t)]$. Such a chart was indicated in *Fig. 14* for $\beta = \frac{1}{5}$. For the sake of comparison, Mr Scott had also prepared a chart for a case of the coefficient of consolidation varying with the degree of consolidation, a case in which computation was otherwise very tedious. It was interesting to note that, in the instantaneous loading case, if the successive values were plotted on such a chart, they eventually became a straight line, that is to say, $u_0(t + \delta t)$

became a straight-line function of both $u_0(t)$ and $[u_2(t) + u_4(t)]$, which again simplified calculations.

The Authors had completed the previously quoted paragraph with a reference to the methods developed in the Paper by Allen and Severn[18] and had stated that the use of those methods precluded "excessive labour." Mr Scott doubted that statement in view of the complexity of the relaxation pattern which those relaxation methods involved. Also, the straightforward computation which iteration methods utilized, which might be

Fig. 14



employed by a person with little knowledge of mathematics, or indeed, of the problem, was forsaken for a more complex solution, demanding initially considerable mathematical dexterity. Mr Scott had also been led to believe that there was a reduction of accuracy with the use of relaxation methods in that instance.

The Authors had presented several comparisons of the iteration method with analytical solutions, showing that good-enough accuracy was obtained. However, in certain circumstances greater accuracy might be desirable and it could be obtained very simply. In *Fig. 2* the iteration process was initiated with pore-water-pressure values of 100, 100, 100 . . . from the surface to the centre of the layer, at $T_v + 0$; then the surface pore-water pressure dropped to 0 at $T_v = \Delta T_v$, and the iteration process was begun. Obviously a step load-increment such as that shown in *Fig. 15*, representing a sudden discontinuity, could not be satisfactorily represented in an iteration procedure, and that gave rise to some error in the solution. If a different initial loading condition, which was demonstrated in *Fig. 16*, was applied and which also, because of the small time interval generally involved in one step compared to the total time of consolidation, represented an instantaneous loading of the soil, a better solution would be obtained. In that procedure, the iteration began with pore-water-pressure values of 50, 100, 100 . . . from the surface to the centre of the layer at $T_v = 0$, and

the surface pore-water pressure was reduced to 0 at $T_v = \Delta T_v$. as shown in *Fig. 17*.

Incidentally, a little easing of the labour involved in that particular solution would have ensued if values of degree of consolidation, and not pore-water-pressure had been used. It could also be shown [14, 19] from a consideration of the errors involved in the iteration expression as derived from Taylor's series, that the choice of $\beta = \frac{1}{6}$ would eliminate one of the errors, and would produce better solutions even than smaller β's and smaller

Fig. 15

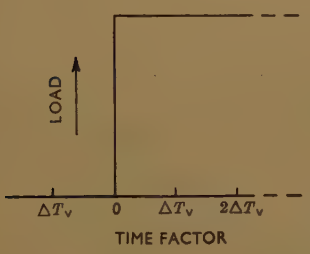


Fig. 16

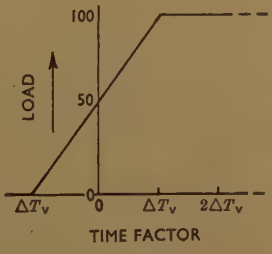
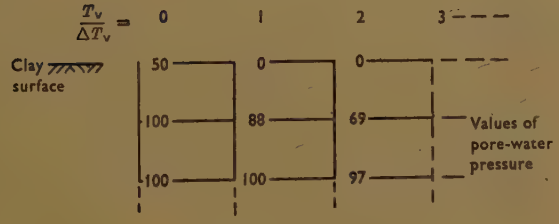


Fig. 17



mesh-spacings. Mr Scott did not know, however, if that applied to all cases. Crandall [20] gave an example.

It might appear to a general reader of the Paper that the non-dimensional functions $\frac{c_v t}{H^2}$ in the one-dimensional, and $\frac{c_v t}{R^2}$ in the radial case were obtained from the analytical solution. That might be clarified by the transformation of the original diffusion equation to a non-dimensional form. Mr Scott gave an example of the one-dimensional equation case, writing equation (1):

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

and putting

$$t = T_v t_0 \quad \ldots \quad (16)$$

$$z = m z_0 \quad \ldots \quad (17)$$

where t_0 denoted some fixed time (that was made clear in a few lines)
T_v ,, a non-dimensional time-proportionality constant (" time factor ")
z_0 ,, some fixed dimension in the physical problem. (For example, z_0 could equal H, the length of the longest drainage path.)
and m ,, a non-dimensional length-proportionality constant ;

then
$$\frac{\partial u}{\partial T_v} = \frac{t_0 c_v}{z_0^2} \frac{\partial^2 u}{\partial m^2}.$$

Putting
$$t_0 = \frac{z_0^2}{c_v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

gave
$$\frac{\partial u}{\partial T_v} = \frac{\partial^2 u}{\partial m^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

That expression could be used as a basis for an iteration equation :
$$u_0(T_v + \delta T_v) = \frac{\delta T_v}{\delta m^2} [u_2 + u_4 - 2u_0] + u_0 \quad . \quad . \quad . \quad (20)$$

which was used as before, equations (16), (17), and (18) being used for the determination of the times of consolidation.

The Authors had found a useful transformation for the hydraulic-fill case in $z = w \tan \alpha$, at the end of the Paper. A similar transformation could be utilized if drain wells were being considered, and that was demonstrated as followed :—

Putting
$$x = \log_e r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

then
$$\frac{\partial x}{\partial r} = \frac{1}{r}$$

and
$$\frac{\partial u}{\partial r} = \frac{1}{r} \cdot \frac{\partial u}{\partial x}$$

The equation for consolidation with radial drainage only was :
$$\frac{\partial u}{\partial t} = c\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial u}{\partial r}\right) \text{(Authors' equation (6))}$$
$$= \frac{c}{r}\left[\frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right)\right]$$
$$= \frac{c}{r^2} \frac{\partial^2 u}{\partial x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

and the appropriate iteration expression became :
$$u_4 = \frac{\beta}{r_0^2} [u_1 + u_3 - 2u_0] + u_0 \quad . \quad . \quad . \quad . \quad . \quad (23)$$

where $\beta = \dfrac{c\delta t}{\delta x^2}$ and r_0 denoted the radius of point O.

The existence of the drain well avoided the difficulty arising at $r = 0$. Values of $\frac{1}{r^2}$ could be tabulated above the appropriate points to be used as the calculation proceeded. In that method, the section was transformed initially, mesh points were chosen at equal intervals, and the iteration process was carried out with less labour than that involved in the use of the Authors' equation. On completion of the work, the section was transformed back to a natural scale. That, it would be seen, resulted in obtaining points closer together near the well itself, which was generally the region of most interest.

Concerning *Fig. 4*, the Authors had made the statement, " In order to extend the solution to a large range of time with the minimum of labour, two cases were worked out. . . ." Mr Scott failed to see how the solution of *two* cases affected in any way the time required to produce an answer up to a given time factor; would the Authors clarify their statement? If, as might be the case, they had used the values $\frac{R}{\delta r} = 5$; $\beta = 0{\cdot}25$ to initiate the solution, and $\frac{R}{\delta r} = 2$, $\beta = 0{\cdot}20$ to continue the solution from small T-values on, then any estimate of errors might be very misleading, since the error was very sensitive to variations in $\frac{R}{\delta r}$ and β. The analysis of error in that case was questionable. In the particular light of the production of two solutions of different mesh-sizes, however, it was pertinent to remark that at any one point in the grid common to the two solutions, Richardson's " h^2-extrapolation " rule [21] could be utilized to give a much more accurate result. It had to be used with care however,[20] because in certain cases, no such improvement would be affected, and the results might be misleading.

With regard to *Fig. 4*, Mr Scott would like to question the legend. According to it, the dashed line represented the analytical solution to the problem involving end drainage only (that is to say, the simple one-dimensional case), and the circles and squares, *together with the solid line through them*, represented the numerical solution to the radial only, and radial and end-drainage cases. Since, in the actual curves, the solid lines rarely touch a plotted point, Mr Scott presumed that those lines were meant to be the analytical solution, and it had been the Authors' intention to attribute those lines to Carshaw and Jaeger. That hypothesis would seem to be confirmed by the last statement in that section.

If the Authors used expression (11), obtained from Carrillo, to achieve an analytical solution to the problem including radial and end drainage, why did they not use it to combine the numerical solutions to the problem, instead of carrying through a very tedious iteration process involving equation (12)? Did they consider they would be compounding the errors?

In *Fig. 6*, a cross indicated a point plotted at $\sqrt{T} = 0{\cdot}175$ and $\bar{U} = 0{\cdot}45$ approximately; was that as much in error as it appeared? Since the other points demonstrated no instability in the calculations, Mr Scott was inclined to suspect an error other than that due to the approximations of the formula. Calculations in such problems could not be checked adequately by other investigators, without all the data being given, and so he would like to recommend that all solutions be accompanied by information about the mesh and the β-value used.

Admittedly it was of practical value to obtain solutions by numerical methods to problems of footings on a settling foundation, but the whole utility of numerical solutions lay in their ability to solve problems with the insertion of fewer assumptions than other methods. For example, in the case of the impermeable circular footing founded on clay with an impermeable (assumed) surface, the Authors had stated that the incorporation of a coefficient of swelling in the iteration process " was not considered worth while." Surely it would not have added much, if anything, to the labour of the problem, already tedious, to have considered, say, a coefficient of swelling of twice the coefficient of consolidation, or any value justified by tests, in order to give weight to any remarks about the size of errors.

In conclusion, Mr Scott emphasized that the validity of the numerical solution depended on the validity of the initial differential equation with its inherent assumptions. However, by using those numerical solutions it was possible to study, with less labour than was caused by the analytical approach, the effect of modifications of the original equation to account for secondary compression, the variation of the coefficient of consolidation in various ways, and thermal and frost problems. In that manner it might be possible to produce eventually theoretical curves in better agreement with those obtained in the laboratory than was, at present, the case. Possibly the greatest advantage would be the use, through iteration methods, of automatic machines to produce the desired solutions with a reduction in human labour.

The Authors, in reply, agreed with Professor Ross and Mr Scott that the use of special values of β often proved advantageous, and they were to be thanked for drawing attention to some graphical procedures which would reduce the labour involved considerably. Values of $\beta > \frac{1}{2}$ had, in fact, been used at an early stage of the work, but owing to the instability mentioned by Dr Severn and Mr Scott, their use had been discontinued.

Some mention had been made in the correspondence of the relaxation method and the Authors felt that its use was only warranted in one-dimensional cases where either a high degree of accuracy was required or extension of the solution to large times was necessary. The complexity of the relaxation pattern appreciably reduced its value, as had been emphasized by Mr Scott, in any but the simplest of problems. The Authors had, however, employed that method in the solution of problem (1) by using the transformation :

$$\eta = 1 - \exp(-\lambda t)$$

which, with equation (1), led to a " residual " F_o given by :

$$F_o \equiv \beta(u_2 + u_4 - 2u_o - (u_1 - u_3)(1 - \eta_o)$$

where

$$\beta = \frac{2c_v\delta\eta}{\lambda\delta z^2}$$

In that way, direct use was made of the boundary condition $u = 0$ when $t = \infty$, and the accuracy in the particular example with $\beta = 1$, $m = 3$ and $\delta\eta = 0{\cdot}2$ had been found to be very high.

In reply to some points raised by Mr Scott, the Authors stated that the areas of the isochrones had been found first by drawing on squared paper as smooth a curve as possible through the individual values of u (at a given value of t) and the area determined simply by counting squares. It was realized that the accuracy of the numerical solution depended, therefore, not only upon the errors introduced by the finite difference approximations but also, subjectively, upon the skill of the operator in drawing and determining the area beneath the isochrones. It had not been the intention of the Authors to consider the question of errors in detail but merely to show that, with suitable choice of the parameter β, and with reasonable care, solutions could be obtained of sufficient accuracy for engineering purposes.

Mr Scott's analysis of the errors in *Fig. 4*, and his supposition regarding the reason for using two cases, were correct. For that same reason, the solution given in *Fig. 6* had been initiated using $\beta = \frac{c_v\delta t}{\delta h^2} = 0{\cdot}25$ and a mesh size $\delta r = \delta z = \delta h = 0{\cdot}125H$, and had been extended using $\beta = 0{\cdot}25$ and $\delta h = 0{\cdot}25H$, the second point of the latter series being that mentioned by Mr Scott. It had been found that the values of $\overline{U}$ corresponding to $n = 1$ or 2 were invariably in error, probably owing to the discontinuity in the surface pore-water pressure at $t = 0$ and were usually rejected. For that reason, that particular point in *Fig. 6* could be disregarded.

In conclusion, the Authors thanked Mr Scott for the references he had given at the end of his contribution.

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Mr Scott recommended references 14, 23, 24, 25, and 27 for initial study, and references 19, 20, and 26 for more advanced investigation.

CORRESPONDENCE

on a Paper published in Proceedings, Part I, May 1953

Paper No. 5879

"Quality Control of Concrete—Its Rational Basis and Economic Aspects" †

by

Niels Munk Plum, M.Dan.Inst.C.E.

Correspondence

Mr L. B. Mercer, of Australia, observed that statistics, at most, could provide only an indication of the probable future general behaviour of concrete when it was assumed that there was no change from the original conditions under which the data were compiled. However, provided the variables involved were appreciated and the improbability of the above assumption was realized, then a knowledge of statistical procedure could furnish assistance in the determination of methods for control.[31, 32, 33] He suggested that perhaps the Paper did not sufficiently stress the importance of sampling and testing variables. Whilst appreciating that errors in testing did not influence the quality of the concrete or disturb the reliability of the structure from which such tests were taken, nevertheless they could indirectly influence both quality and economy through setting an incorrect "target" strength. Mr Mercer considered that quality control of concrete through statistical analysis should accordingly make separate provision for the variables of testing, which would be different for different laboratories and conditions; he further suggested that the cement variable should also be separated from the other factors involved in control of concrete production.

† Proc. Instn Civ. Engrs, Pt I, vol. 2, p. 311 (May 1953).

[31] L. B. Mercer, "Concrete Strength Variation—60 Contributory Causes." *Building and Construction*, 28 March and 5 April, 1950.

[32] L. B. Mercer, "Concrete Quality Control—Fallacious Application of Statistical Analysis." *Commonwealth Engineer*, December 1950.

[33] L. B. Mercer, "Concrete Mix Design." Research Bulletin No. 2, Melbourne Technical College Press, 1953.

For instance, if V_c represented the coefficient of variation for the compression strength properties of the cement, excluding the cement testing variables; V_p represented the coefficient of variation for the plant and aggregate variables, including batching and mixing with control of variations in the water/cement ratio and workability; and V_t represented the coefficient of variation for the sampling and testing of concrete specimens, then V, the coefficient of variation for the complete concrete production and testing, was given by the formula:

$$V = \sqrt{(V_c)^2 + (V_p)^2 + (V_t)^2}.$$

For example, if V_c varied from 4 per cent, in the case of a mixing plant drawing cement from one reliable factory, to 9 per cent where the cement was drawn from, say, several factories operating with different control limits; V_p varied from 7 per cent for concrete batching and mixing plants using uniform aggregate and equipped for accurate weigh-batching operations with strict control of the water, to 14 per cent for variable materials and indifferent control; and V_t varied from 4 per cent for strict concrete testing to 7 per cent for indifferent testing, then V might vary between:

$$V = \sqrt{(4)^2 + (7)^2 + (4)^2} = 9 \text{ per cent}$$

$$\text{and } V = \sqrt{(9)^2 + (14)^2 + (7)^2} = 18 \text{ per cent}$$

It was possible, on that basis, to provide for variations in the cement from time to time, to adjust for different plants operating under various conditions in several locations, and to modify the figures according to any changes in the efficiency of the testing laboratory. Provision could readily be made for major variations, such as a change in the type of cement or aggregate, by adjusting the " target " average.

The recommended procedure for the design of mixes was thus to determine V_c, V_p, and V_t, experimentally, calculating V (overall coefficient) and checking for the complete concrete production operation by experiment. Changes in V_c, V_p, and V_t would have to be anticipated, with the aid of experience and prior experimental knowledge, so that the modified V could be obtained and the " target " average adjusted accordingly. The average had also to be revised to provide allowance for radical changes in " level." Thus, if a change was made to a cement which gave 500 lb. per square inch variation in average strength under otherwise similar conditions to those upon which the original concrete average was based, then the original concrete " target " average had to be modified accordingly for such variation.

It was contended that it was possible, with the above amplified application of statistical analysis, to secure increased economy with safety.

Mr E. R. Giles noted that the Author had sought to show the desirability of sampling in such a way that variation in compressive strength of cubes could be attributed separately to " within batch " or " between batch " causes, in order that the means of achieving a more " economical

homogeneity " might be discovered. One had to bear in mind, however, that compressive strength was a complicated quality characteristic which itself was amenable to analysis, and which was not capable of interpretation to the degree of accuracy required for differentiation between the sources of variations unless a very large number of samples was taken for continuous checking. Yet, of the factors affecting the strength of cubes, only two were relevant to the strength of the concrete on the job, the cement strength and the water/cement ratio.

For the former, a probable value was obtainable direct, initially from the supplier and later from tests as the job proceeded. It would seem useful to measure the latter direct as well, since a continuous record of water/cement ratios would permit the calculation of a standard deviation for them—that would be difficult to infer from the cube results because the distribution of compressive strengths was not a Gaussian curve for a random distribution of water/cement ratios, as could be seen from *Fig. 6*. In that Figure, the broken line depicted a Gaussian distribution

Fig. 6

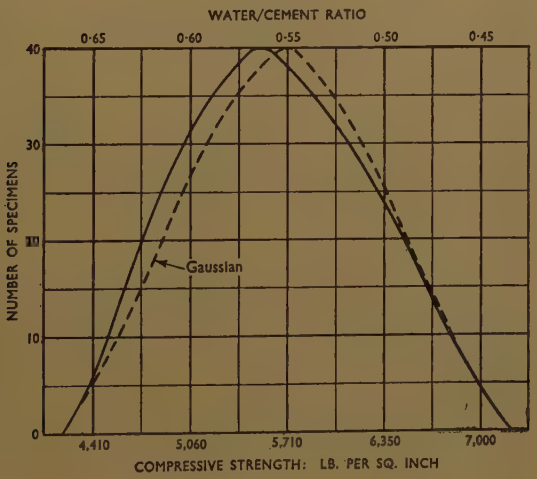
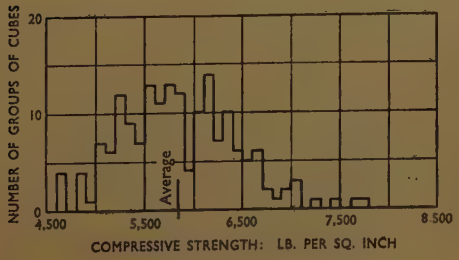


Fig. 7



about a water/cement ratio of 0·55, and the full line a distribution derived from it by interpreting the various ratios as strengths for a standard cement strength and plotting those strengths on a linear scale between the extremes of the Gaussian curve. Asymmetry of that kind was noticeable in *Fig. 7*, which was a histogram of 28-day strengths from a current contract covering about 80,000 cubic yards of concrete, and it was dependent to some extent on the strength level. Similarly, asymmetry could be observed in *Figs 45* and *46* of Airport Paper No. 1.[34]

A method of measuring water/cement ratio direct was given in B.S.1881 and could be used for continuous checking. Although the results of those tests would be themselves inaccurate they would have the virtue that the figure to which they approximated would be one which was strictly relevant to the problem in hand. The engineer could criticize himself on the basis of the result without having to bear in mind irrelevant factors which had served as a means of producing it. Mr Giles emphasized that, once a mix had been designed, the contractor was not aiming to produce a strength but to achieve consistent batching.

Did the Author consider that site control by the method outlined above would be preferable to the present method of control by cube samples in seeking the cause of lack of homogeneity ?

The Author, in reply, expressed his appreciation of Mr Mercer's valuable contribution.

As soon as the importance of variation in concrete strength on quality was realized, the next step was to analyse the different sources of the variation, and to determine which contribution was of the greatest influence ; reduction of that specific contribution could then lead to a reduction of the composite coefficient.

Mr Mercer had distinguished between the variations caused by cement, mixing-plant, and testing. Any one of them was a composite coefficient of variation, with several contributions, and by localizing one major component it was possible to obtain a great overall reduction of the variation.

The Author doubted whether V_c, as determined from test specimens of mortar, gave a true indication of the variation in concrete strength, which arose from variation in properties of the cement ; at least, that was the case when the mortar-specimens were manufactured using a standard sand with a very steep gradation curve.

The Author did not agree that specifications for the control of concrete quality should make special provision for errors of testing arising from the variation of laboratory conditions. For instance, the statistical analysis by which the most economical homogeneity was determined (see *Fig. 3*) might yield quite misleading results if the component of the standard deviation arising from testing errors had not been eliminated from the standard deviation by which the concrete was characterized.

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[34] See reference 2 in the Paper.

Mr Giles had questioned whether the Author considered that control of cement strength and water/cement ratio was preferable to control by cube samples when seeking the cause of lack of homogeneity.

In reply, the Author summarized his experience thus :—

(1) Cube tests were of little value, first, because of the differences in mean strength and standard deviation between the cubes and the structure (*cf.* pp. 326–330 of the Paper) and secondly, because the results were at least one week delayed.

(2) It was, therefore, natural, to attempt control of the qualities of the concrete components prior to mixing and the mixing proportions simultaneously with the mixing, so that corrections could be made whilst the mix was still in the mixer.

Mr Giles's suggestion was, therefore, a step in the right direction. The Author fully agreed to the earliest possible control of the cement quality, but he doubted whether control of the water/cement ratio was a sufficient supplement.

It should be emphasized that concrete strength was not governed by the water/cement ratio but by the water-plus-voids/cement ratio, which meant that the water/cement approximation held good only when the volume of voids was negligible, that was, when the workability corresponded to the method of compaction and that, therefore, also had to be controlled.

Experience showed beyond doubt that the compactness was not always satisfactory, and that was one of the reasons why the Author had recommended continuous control of the workability. The second reason was that the Author considered direct measurement of the water/cement ratio or the absolute water content very dubious and delaying, whereas indirect measurement of the water content via the workability or consistency measurements could be made more correctly and speedily.

This indirect measurement rested on the assumption of constant grading of all solids, discussed at length on pp. 330–331 of the Paper.

OBITUARY

ALFRED BROWN ERNEST BLACKBURN, C.B.E., B.Sc., who died on the 22nd September, 1953, was born at Walsall, Staffordshire, on the 8th January, 1876. He was educated at Solihull Grammar School, Warwickshire, and at Mason University College, Birmingham.

In 1895 he entered upon a 3 years' pupilage with Mr A. Ashton Hill, M.I.C.E., and on its completion he became a Junior Engineer with the South Staffordshire Waterworks Company.

In 1901 he joined the staff of the Sunderland and South Shields Water Company as Engineer and General Manager, and later was made a Director. He retired in 1949, but retained his seat on the Board.

Mr Blackburn served on several advisory committees, both on water engineering and mining subsidence. These included the Water Power Resources Committee (1919–1921) and the Ministry of Health Advisory Committee on Water (1922–1937). In 1924 he gave evidence on behalf of the British Waterworks Association and the Water Companies Association before the Royal Commission on Mining Subsidence, and later prepared evidence for Birmingham Corporation in connexion with the Elan Valley works.

In recognition of these services he was made a Commander of the Order of the British Empire.

Mr Blackburn was elected an Associate Member of the Institution in 1901, and was transferred to the class of Member in 1920. He was also a Past-President and Member of Council of the Institution of Water Engineers, a Past-President of the British Waterworks Association, and Chairman of the Northern J.I.C. Water Industry.

As a Student he was awarded a Miller Prize for his Paper on " Engineering Education from a Student's Standpoint," and subsequently wrote several Papers on water engineering.

He leaves three sons, a daughter, and two step-daughters.

JAMES WILLIAMSON, C.B.E., who died on the 18th August, 1953, was born at Holytown, Lanarkshire, on the 5th April, 1881. He was educated at the Royal Technical College, Glasgow, and at Glasgow University, and served a 4-year apprenticeship with the Engineer to the Middle Ward of the County of Lanark.

His first post was that of Assistant Engineer with the firm of Formans and McCall, which he took up in 1903; he became a Chief Assistant in 1907. His early work was connected mainly with railway construction, but from 1908 to 1912 he was in charge of varied works, including a drainage system for Neilston, Renfrewshire; extensions to Ardrossan Harbour and graving dock; structural designs for office buildings in Glasgow, and

extensions to the Savoy Theatre, Glasgow, and to the Institution of Mechanical Engineers, London.

In 1912 and 1913, after a short period as assistant to Baker and Hurtzig, Consulting Engineers, of Westminster, Mr Williamson gained further experience in structural design and manufacture as assistant to the Chief Engineer of the Cleveland Bridge and Engineering Co., Darlington. The work included the design of large material-handling travelling gantries for the erection yard, deep pier foundations, large-span railway bridges, and an early arch-bridge proposal for Sydney Harbour.

He returned to Formans and McCall as Chief Assistant from 1913 to 1915, and was responsible for the design of structural work for a new passenger station at Aberdeen, and other large buildings. At the beginning of World War I, he carried out extensive tacheometric surveys for Nobel's cordite factory at Ardeer, Scotland, and designed an 80-mile railway system for H.M. explosives factory at Gretna. He was also employed at the head office of the Explosives Department, Westminster, on the preparation of lay-out plans for the Gretna factory. In 1915, he was transferred to the staff of S. Pearson & Son, Ltd, and was made head of the Civil Engineering Department for the acids and guncotton section of the Gretna factory.

He later occupied, for a short while, a similar position at the Henbury Explosives Factory, near Bristol, but in 1917 he joined the Royal Engineers as a lieutenant, and was engaged at the War Office in design work for Channel train ferries, elevated concrete water-tanks, and other reinforced-concrete structures. In 1918, he was seconded to John Vermehr for work on reinforced-concrete craft. He later became Chief Designer, and then Chief Engineer, with this firm.

Following 2 years in private practice, he joined Sir Alexander Gibb & Partners, Westminster, as Chief Engineer, and remained with this firm until 1936. Major works for which he was responsible during this period included Barking Power Station—the largest steam power station in Britain; the Gander Valley Water Power and Paper Scheme, Newfoundland, and the Uhl River Water Power Scheme, Mandi State, India, for which he prepared reports; the Salt River Steam Power Station, Cape Town, for which he prepared designs and specifications; the new Strand Market Wharf and Transit Sheds, Rangoon, and reconstruction of the Sule Pagoda Wharf; and the Galloway Water Power Scheme, with which he was connected from its very beginnings.

After a further short period in private practice (1937–38), Mr Williamson became Director and Chief Engineer of the firm of Sir William Arrol & Co. Ltd. Again he was responsible for much varied work, including the design and construction of bridges, factory buildings, and shipyard installations.

In 1942, he was appointed a member of the Cooper Committee, established to report on hydro-electric development in Scotland, and in 1943 he joined the North of Scotland Hydro-Electric Board. He resigned his post

with Sir William Arrol & Co. Ltd. in 1944 to undertake further hydro-electric work as a private consultant in Glasgow. The firm of James Williamson and Partners was constituted in 1947.

He was largely responsible for the design and construction of the Loch Sloy, Tummel-Garry, and Shira power schemes, and it was for this work that he received the C.B.E. in the 1953 New Year Honours List.

Mr Williamson was elected an Associate Member of the Institution in 1907, and transferred to the class of Member in 1923. He was the Author of several technical books and Papers, including those presented to the Institution on "Jetty and Pump-House Works at Barking Power-Station," [1] and on "Considerations of Flow in Large Pipes, Conduits, Tunnels, Bends, and Siphons," [2] for which he was awarded a Telford Premium and a Crampton Prize, respectively, by the Council of the Institution.

He is survived by his wife and four daughters.

SIR JOHNSTONE WRIGHT, who died on the 19th July, 1953, was born at Dunning, Perthshire, on the 22nd January, 1883. He received his early education at private school, and later studied at Perth Academy and at the Royal Technical College, Glasgow. He served a 5-year apprenticeship under D. Stewart & Co., Engineers, of Glasgow.

He was mainly concerned throughout his career with the construction, maintenance, and operation of power plant and mains, and in later years with the management of electricity undertakings.

His first appointment was that of Assistant Testing Engineer with the British Electric Plant Co., in 1903. From 1906 to 1919, he was employed by the North East Coast Power Companies, as Construction and Mechanical Engineer, and later as Power Station Operation Engineer.

He was Deputy City Electrical Engineer of Bradford from then until 1922, when he became Chief Electrical Engineer and General Manager of Belfast Corporation Electricity Undertaking.

In 1927, he was appointed Deputy Chief Engineer, and in 1932, Chief Engineer, of the Central Electricity Board. From 1944 until his retirement in 1947, he was General Manager to the Board.

Sir Johnstone, who was knighted in 1943, was elected a full Member of the Institution in 1934. He was President of the Institution of Electrical Engineers for the 1939–40 Session, and in 1944 was nominated to serve on the Engineering Joint Council as representative of that Institution. He was the Author of several Papers on electrical engineering.

He is survived by Lady Wright and his two daughters.

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[1] J. Instn Civ. Engrs, vol 11, p. 451 (Apr. 1939).

[2] Min. Proc. Instn Civ. Engrs, vol. 226, p. 135 (1927–28, Pt 2).